

# Metabolic flux analysis for modeling dynamic MRS data

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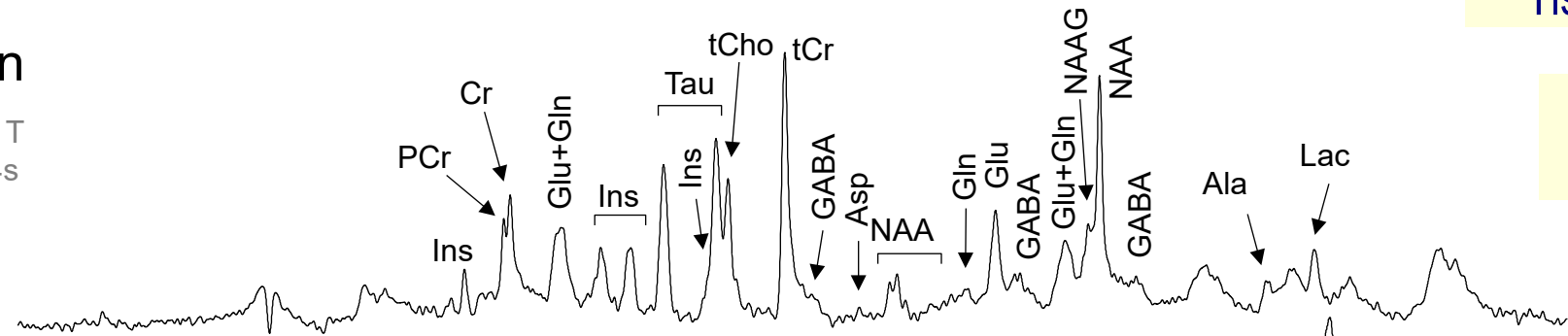
## *Today's menu:*

- ✓ Dynamic magnetic resonance spectroscopy (MRS)
- ✓ Metabolic tracers in MRS
- ✓ Mathematical models of metabolism
- ✓ Metabolic flux analysis (MFA) *in vivo*

# $^1\text{H}$ MRS for metabolite profiling *in vivo*

## Mouse brain

SPECIAL @ 14.1 T  
TE=2.8 ms, TR=4s



Tissue specificity

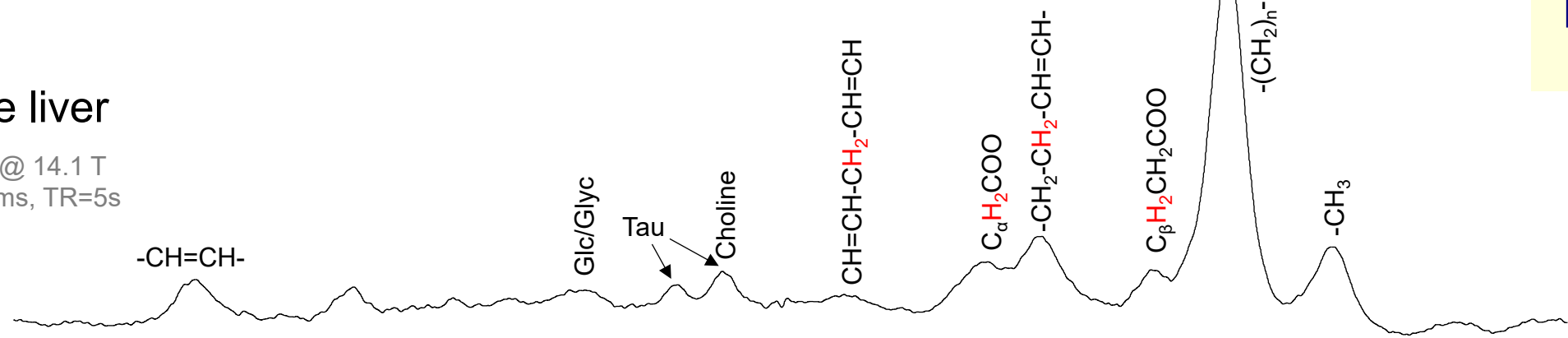
Degeneration or regeneration

Metabolic status

Tissue development and differentiation

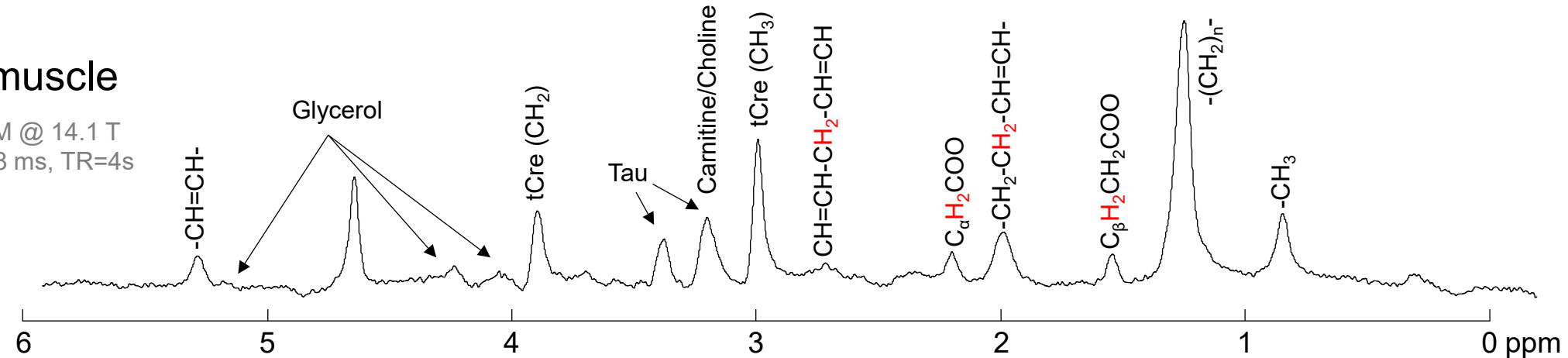
## Mouse liver

STEAM @ 14.1 T  
TE=2.8 ms, TR=5s



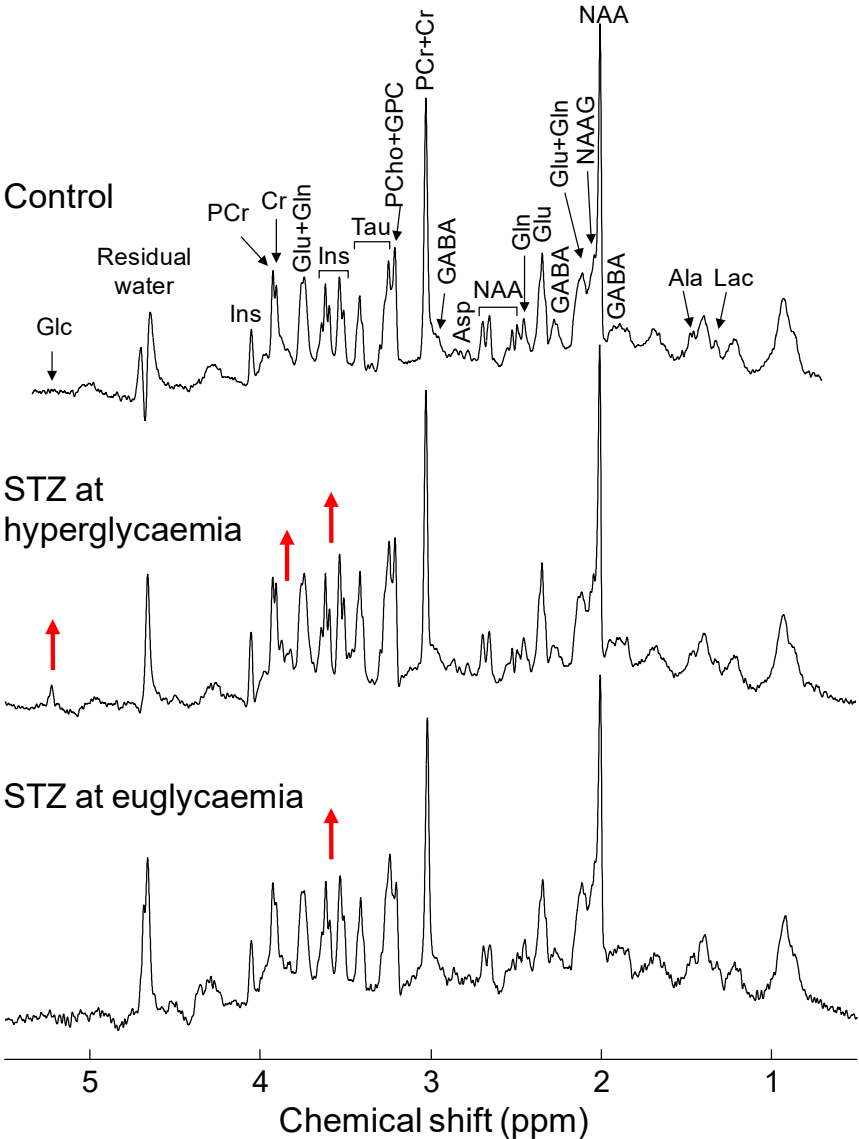
## Rat muscle

STEAM @ 14.1 T  
TE=2.8 ms, TR=4s



6 5 4 3 2 1 0 ppm

# Glucose in $^1\text{H}$ MRS of the rat brain



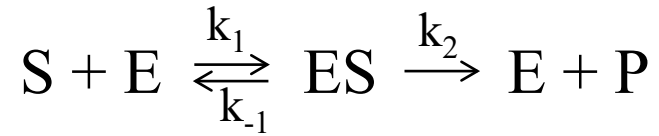
## $^1\text{H}$ MRS of the rat hippocampus *in vivo* at 9.4 T

Red arrows stand out the increase in glucose and *myo*-inositol signals in the streptozotocin (STZ) model of diabetes.

Duarte *et al.*, (2009) *J Neurochem* 111:368

# Recalling what you know on enzyme kinetics:

## Single substrate Michaelis-Menten kinetics



$$v = \frac{d[P]}{dt} = k_2 [ES]$$

$$\frac{d[ES]}{dt} = k_1 [E][S] - (k_{-1} + k_2)[ES] = 0$$

$$\Rightarrow [ES] = \frac{[E]_t [S]}{\frac{k_{-1} + k_2}{k_1} + [S]}$$

$$\Rightarrow v_0 = \frac{k_2 [E]_t [S]}{K_M + [S]}$$

$$v_0 = \frac{V_{\max} [S]}{K_M + [S]}$$

**Rapid equilibrium assumption:**

Rate limiting step is  $ES \rightarrow E + P$  ( $k_2 \ll k_{-1}$ )

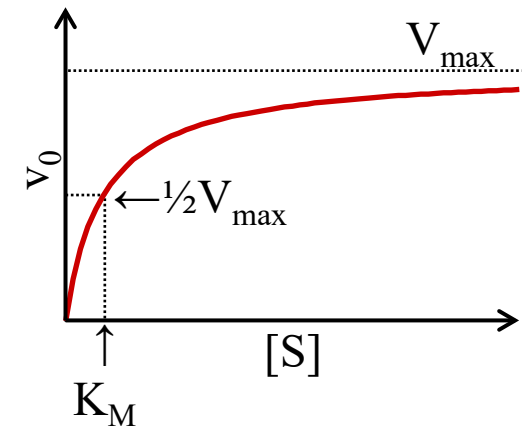
Brown (1902) *J Chem Soc* 81:373

$[S] \gg [E]$ , implying  $[S]_0 = [S]$

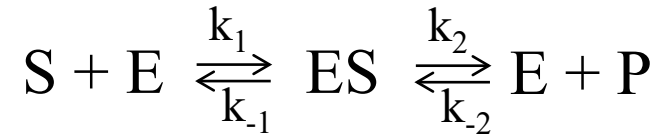
Michaelis & Menten (1913) *Biochem Z* 49:333  
based on work of Henri (1903)

**Steady-state assumption:**  $[ES]$  is constant

Brigs & Haldane (1925) *Biochem J* 19:338



# Reversible Michaelis-Menten kinetics



$$v_0 = \frac{\frac{V_{\max}^f}{K_M^S} [S] - \frac{V_{\max}^r}{K_M^P} [P]}{1 + \frac{[S]}{K_M^S} + \frac{[P]}{K_M^P}}$$

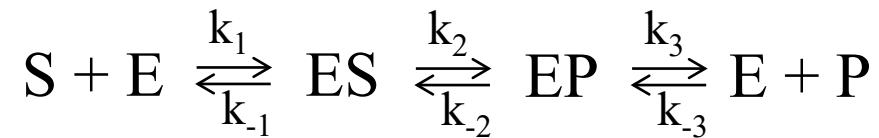
$$V_{\max}^f = k_2 [E]_t$$

$$V_{\max}^r = k_{-2} [E]_t$$

$$K_M^S = \frac{k_2 + k_{-1}}{k_1}$$

$$K_M^P = \frac{k_2 + k_{-1}}{k_{-2}}$$

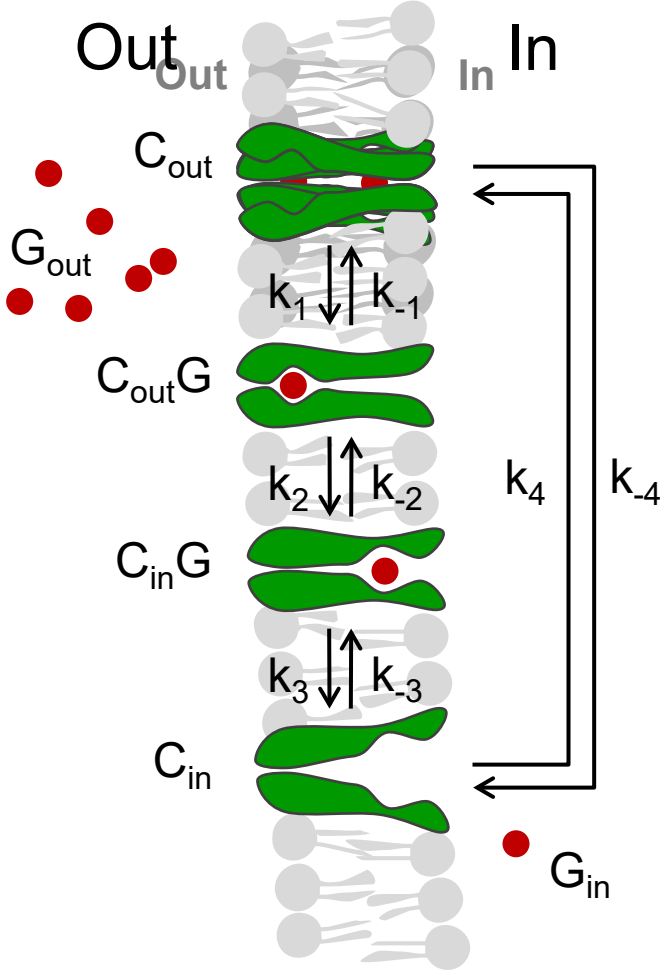
# Enzyme kinetics with multiple intermediates



easy with the King-Altman method

King & Altman (1956) J Phys Chem 60:1375

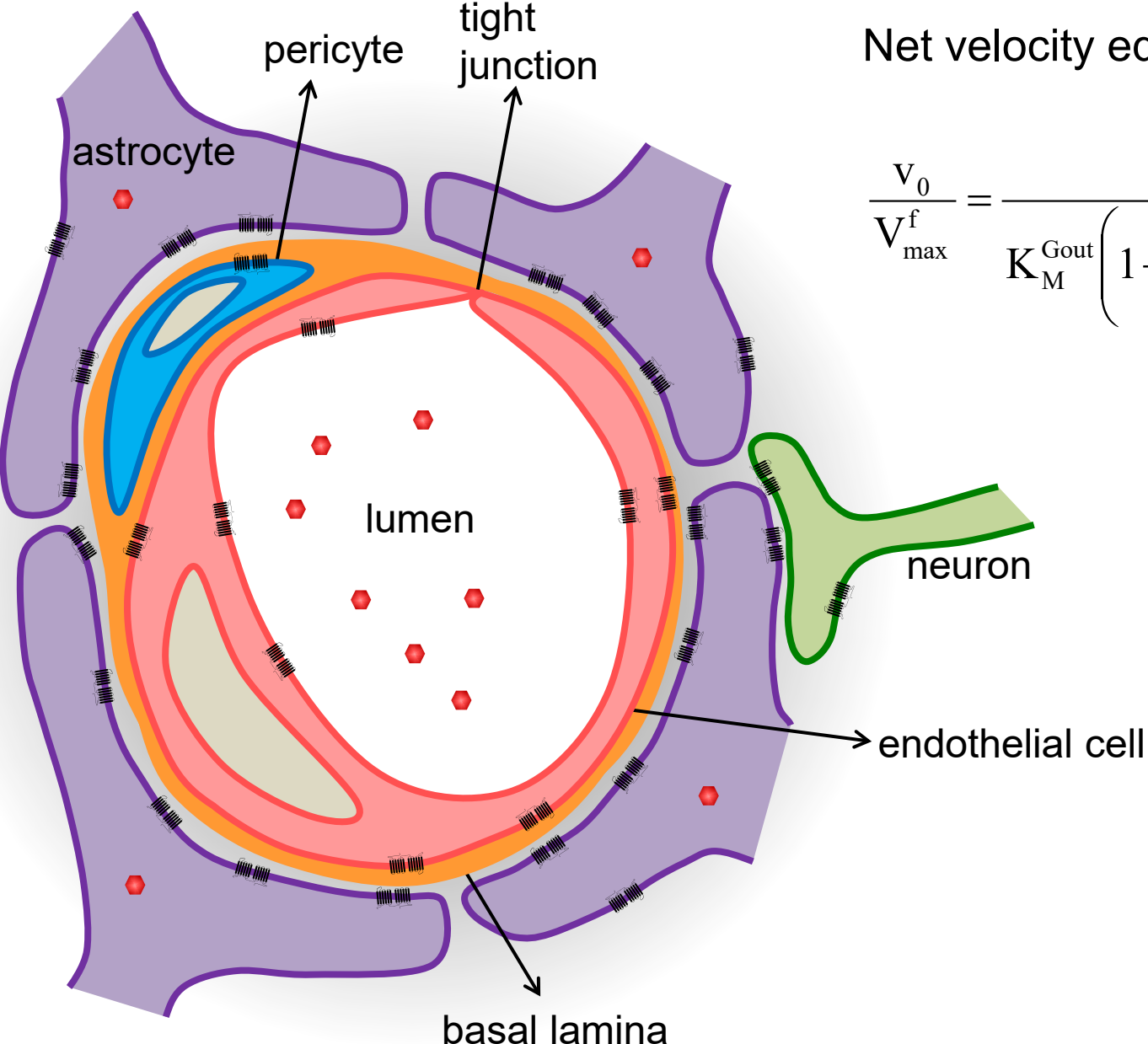
# Glucose transport: 4-state conformational model



Net velocity equation:

$$\frac{V_0}{V_{\max}^f} = \frac{G_{\text{out}} - \frac{G_{\text{in}}}{K_{\text{eq}}}}{K_M^{\text{Gout}} \left( 1 + \frac{G_{\text{in}}}{K_M^{\text{Gin}}} \right) + G_{\text{out}} \left( 1 + \frac{G_{\text{in}}}{K_{\text{ii}}} \right)}$$

# The blood-brain-barrier



Net velocity equation:

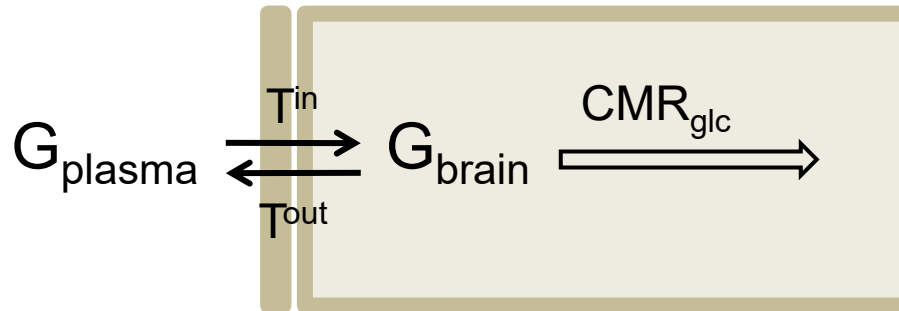
$$\frac{v_0}{V_{\max}^f} = \frac{G_{\text{out}} - \frac{G_{\text{in}}}{K_{\text{eq}}}}{K_M^{\text{Gout}} \left( 1 + \frac{G_{\text{in}}}{K_M^{\text{Gin}}} \right) + G_{\text{out}} \left( 1 + \frac{G_{\text{in}}}{K_{\text{ii}}^{\text{Gin}}} \right)}$$

# Model of brain glucose dynamics requires model simplification

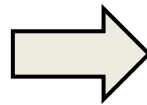
## Assumptions:

- BBB is a single membrane
- Glucose transport is symmetric
- Glucose consumption only occurs in brain parenchyma

Duarte et al. (2009) *Front Neuroenerg* 1:6



$$\frac{dG_{\text{brain}}}{dt} = T^{\text{in}} - T^{\text{out}} - \text{CMR}_{\text{glc}}$$



$$\frac{dG_{\text{brain}}}{dt} = \frac{T_{\text{max}} \left( G_{\text{plasma}} - \frac{G_{\text{brain}}}{V_d} \right)}{K_t + \frac{G_{\text{brain}}}{V_d} + G_{\text{plasma}} \left( 1 + \frac{G_{\text{brain}}}{V_d K_{ii}} \right)} - \text{CMR}_{\text{glc}}$$

## Net velocity equation:

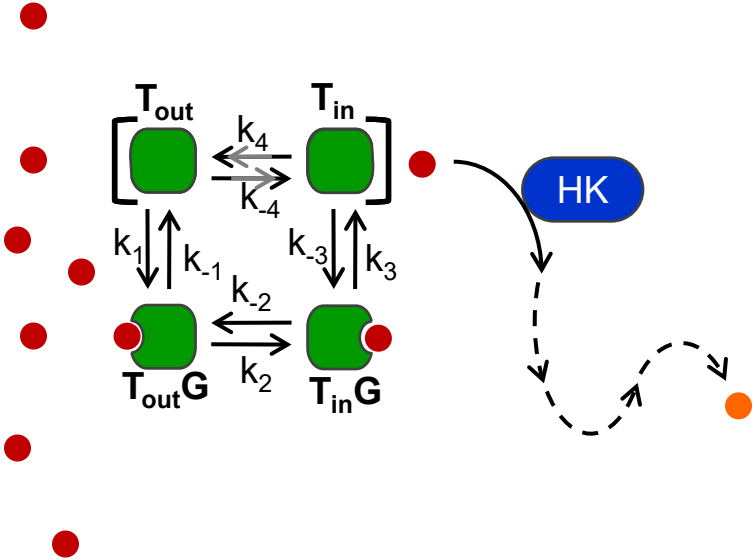
$$\frac{V_0}{V_{\text{max}}^f} = \frac{G_{\text{out}} - \frac{G_{\text{in}}}{K_{\text{eq}}}}{K_M^{G_{\text{out}}} \left( 1 + \frac{G_{\text{in}}}{K_M^{G_{\text{in}}}} \right) + G_{\text{out}} \left( 1 + \frac{G_{\text{in}}}{K_{ii}} \right)}$$

facilitated diffusion:  $K_{\text{eq}} = 1$

Cuppoletti & Segel (1975) *J Theor Biol* 53:125

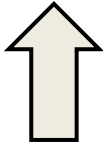
Further reading: **Cerebral glucose delivery, transport and metabolism: Theory and modeling using four, three, and two tissue compartments.** Seidemo et al. *J Cereb Blood Flow Metab* 2025

# Fast isomerisation of unloaded carrier (Kii >> Gbrain)



**steady-state** →

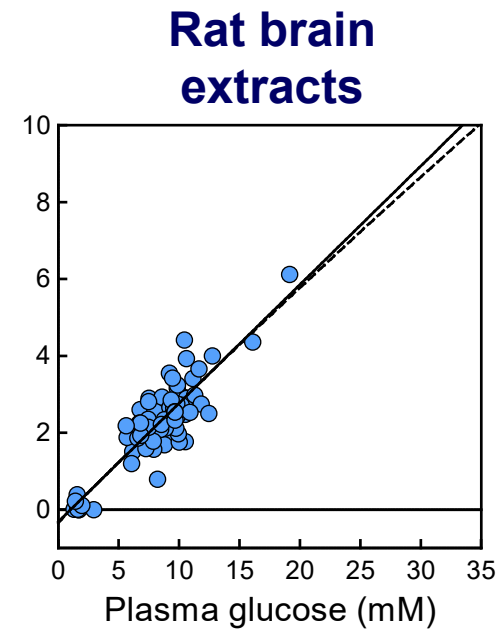
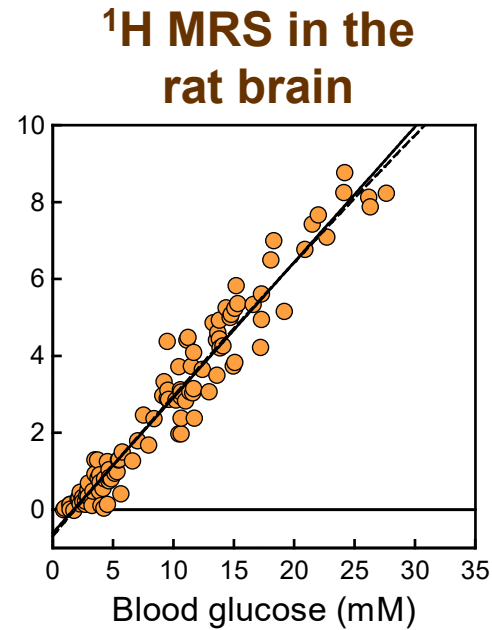
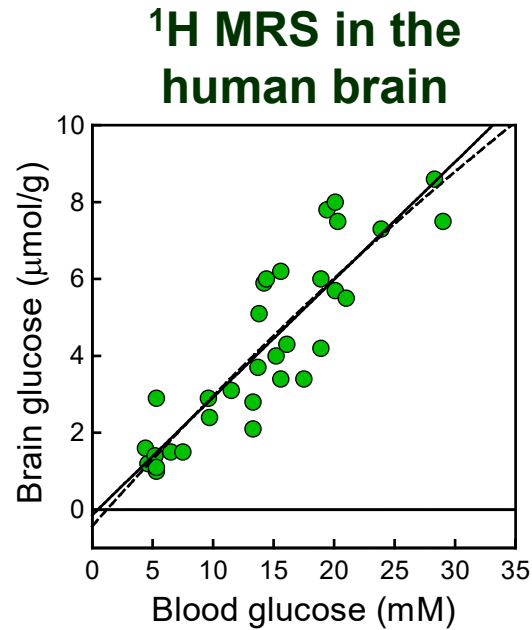
$$G_{\text{brain}} = V_d \frac{\left( \frac{T_{\text{max}}}{\text{CMR}_{\text{glc}}} - 1 \right) G_{\text{plasma}} - K_t}{\frac{T_{\text{max}}}{\text{CMR}_{\text{glc}}} + 1}$$



$$0 = \frac{T_{\text{max}} \left( G_{\text{plasma}} - \frac{G_{\text{brain}}}{V_d} \right)}{K_t + \frac{G_{\text{brain}}}{V_d} + G_{\text{plasma}}} - \text{CMR}_{\text{glc}}$$

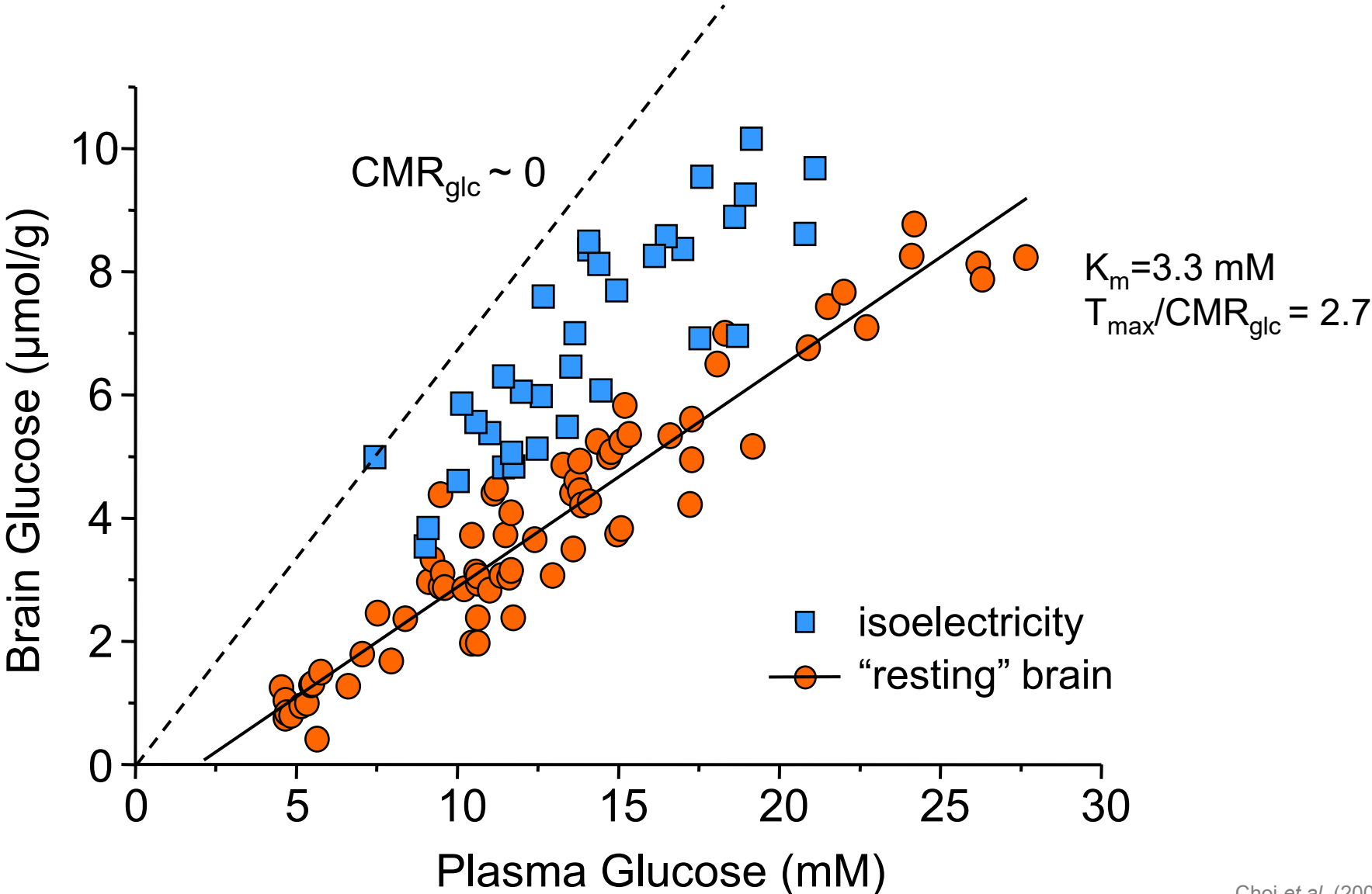
reversible Michaelis-Menten model

# Analysis of steady-state brain glucose transport

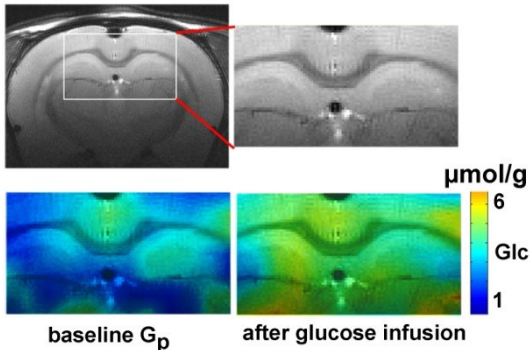


	Rev. MM	4-state conf.	Rev. MM	4-state conf.	Rev. MM	4-state conf.
$T_{\text{max}}/\text{CMR}_{\text{glc}}$	$2.3 \pm 0.2$	$2.7 \pm 1.2$	$2.7 \pm 0.1$	$2.9 \pm 0.3$	$2.3 \pm 0.1$	$2.4 \pm 0.4$
$K_t$ (mM)	$0.6 \pm 2.0$	$2.1 \pm 5.2$	$2.9 \pm 0.5$	$3.5 \pm 1.0$	$1.4 \pm 0.7$	$1.5 \pm 1.1$
$K_{ii}$ (mM)		$50.8 \pm 161.5$		$105.3 \pm 151.7$		$145.9 \pm 776.4$

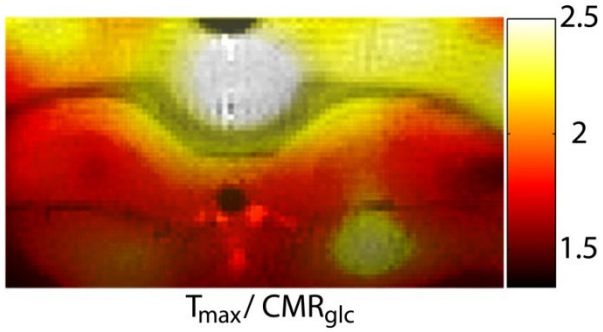
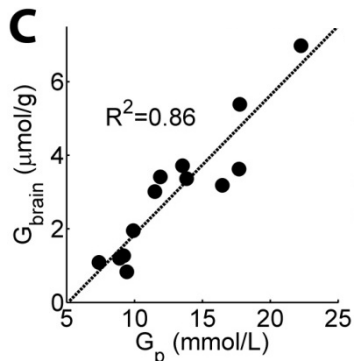
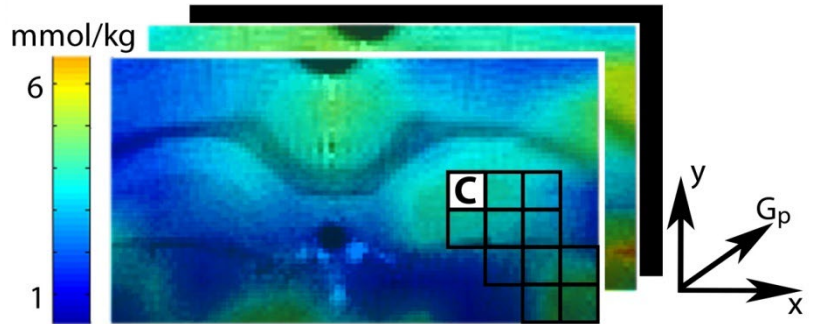
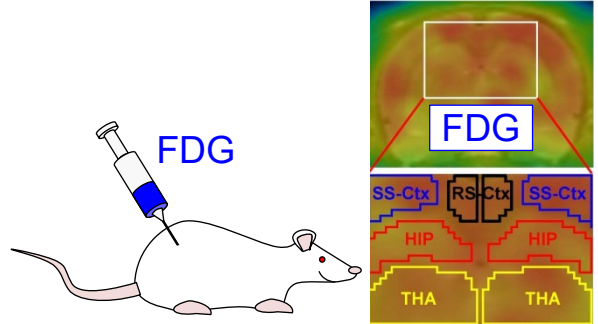
# Reduced metabolism increases brain glucose content



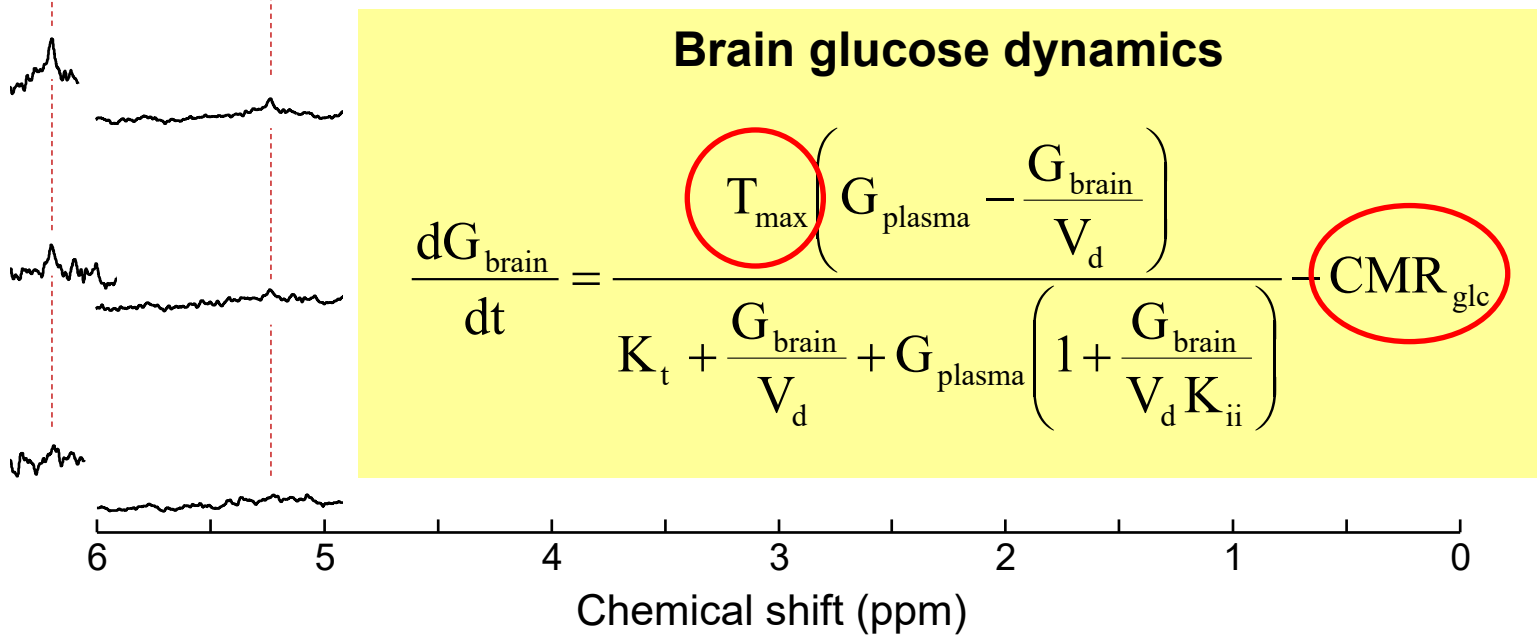
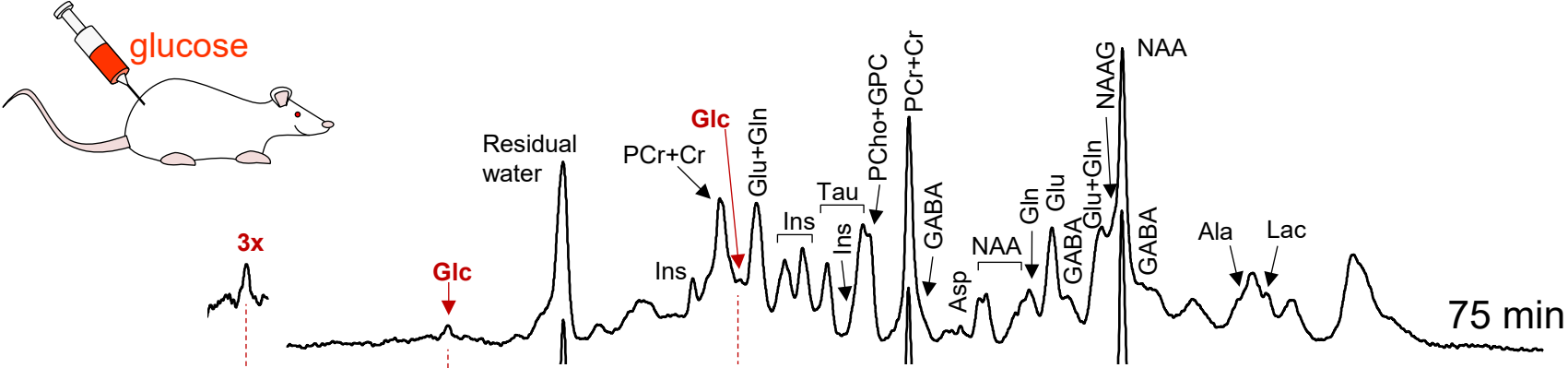
# Spatial mapping of brain glucose using spectroscopic imaging methods



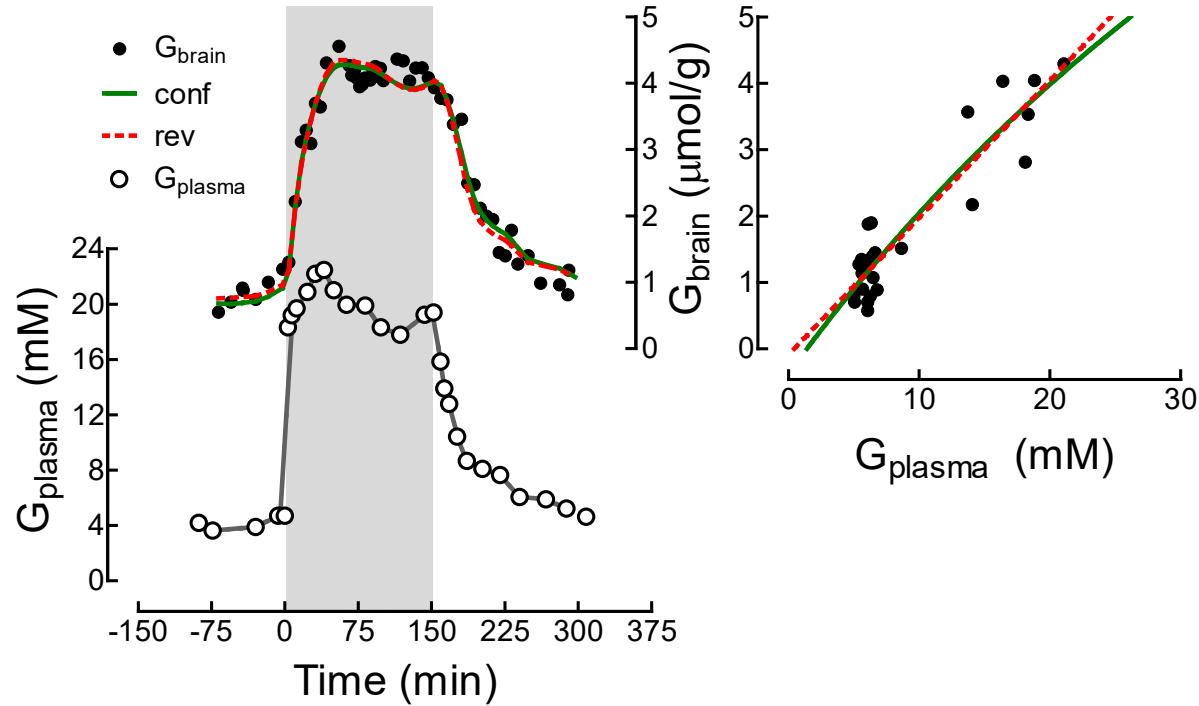
Comparable to PET  
with  $^{18}\text{F}$ -Fluorodeoxyglucose (FDG)



# Typical $^1\text{H}$ MRS at 9.4 T during the time course of glucose infusion



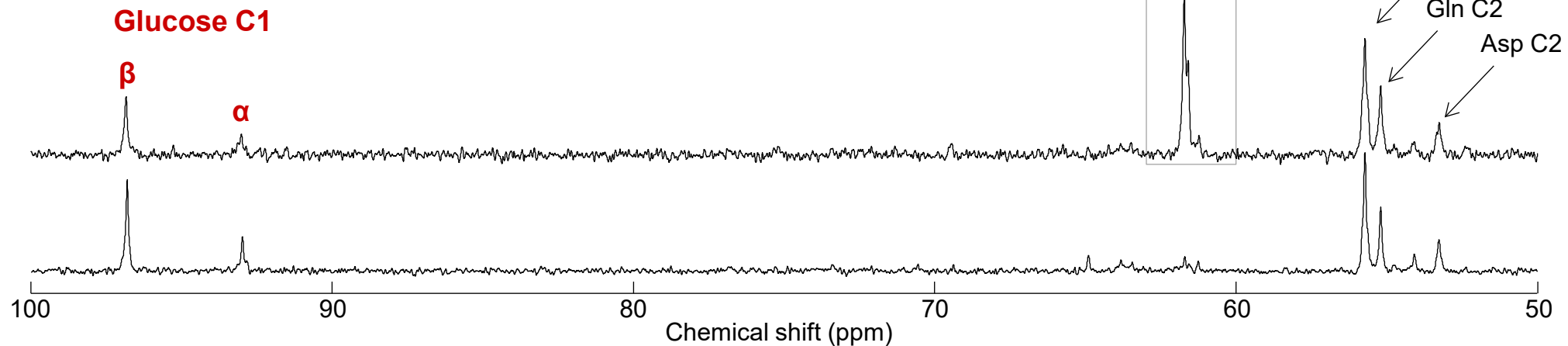
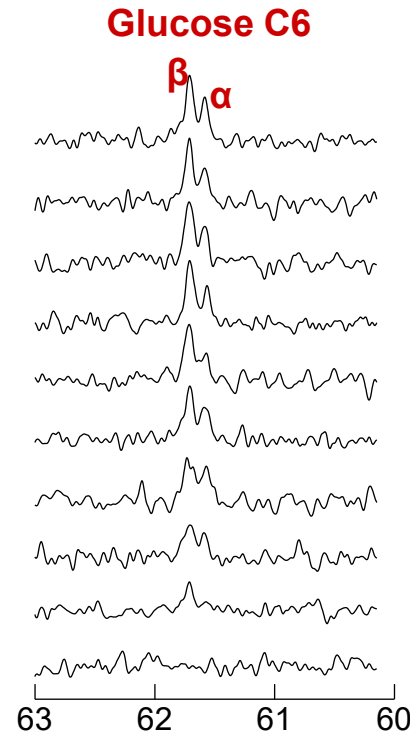
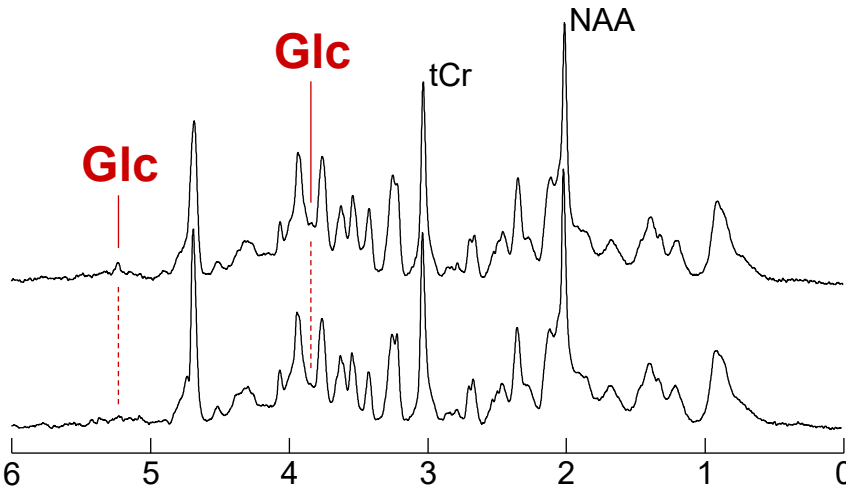
# Dynamic analysis of glucose transport across the BBB



	Conformational model	Reversible MM model
$T_{\text{max}}$	$1.03 \pm 0.18$	$0.99 \pm 0.18$
$\text{CMR}_{\text{glc}}$	$0.52 \pm 0.09$	$0.59 \pm 0.11$
$K_t$	$1.43 \pm 0.33$	$0.64 \pm 0.27$
$K_{\text{ij}}$	$50.3 \pm 16.0$	

	Steady-state measurement	Dynamic measurement
<b>Rev. MM model</b>	$T_{\text{max}}/\text{CMR}_{\text{glc}}$	$1.73 \pm 0.08$
	$K_t$	$0.32 \pm 0.66$
<b>Conf. model</b>	$T_{\text{max}}/\text{CMR}_{\text{glc}}$	$2.06 \pm 1.21$
	$K_t$	$1.49 \pm 4.32$
	$K_{\text{ij}}$	$26.4 \pm 96.8$

# Glucose transport by $^{13}\text{C}$ MRS – some advantages



# Glucose transport by $^{13}\text{C}$ MRS – some advantages

$^{12}\text{C}$  has zero net spin

not detectable by NMR

$^{13}\text{C}$  tracing  
(think of FDG-PET, but add chemical specificity)

$^{13}\text{C}$  is a stable isotope with 1.1% of natural abundance (of C atoms)  
gyromagnetic ratio is 1/4 that of  $^1\text{H}$

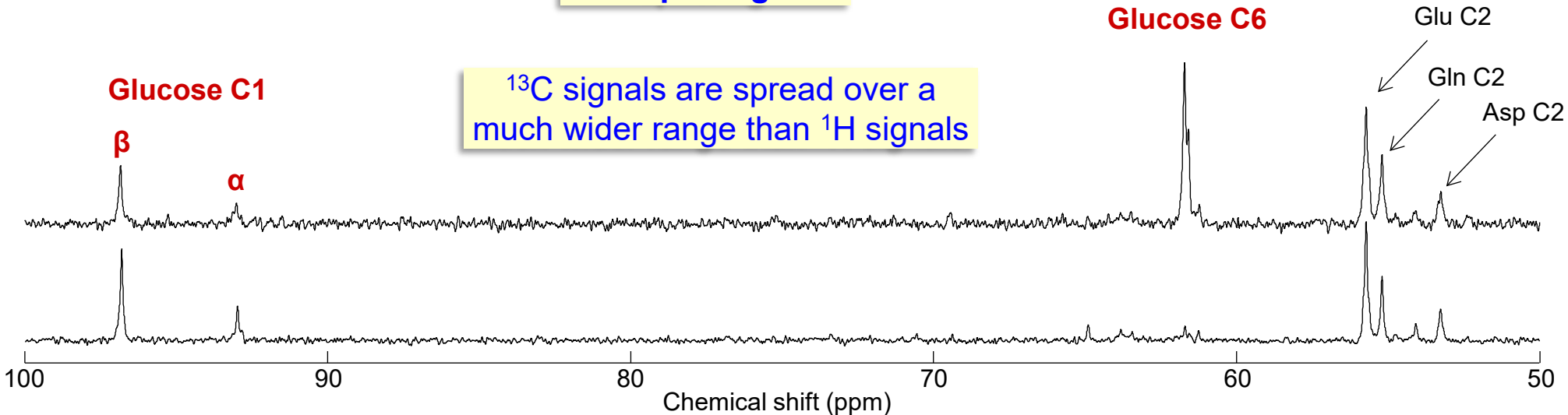
Reduced sensitivity  
(~400 times less sensitive than  $^1\text{H}$ )

Sensitivity boost with hyperpolarized NMR

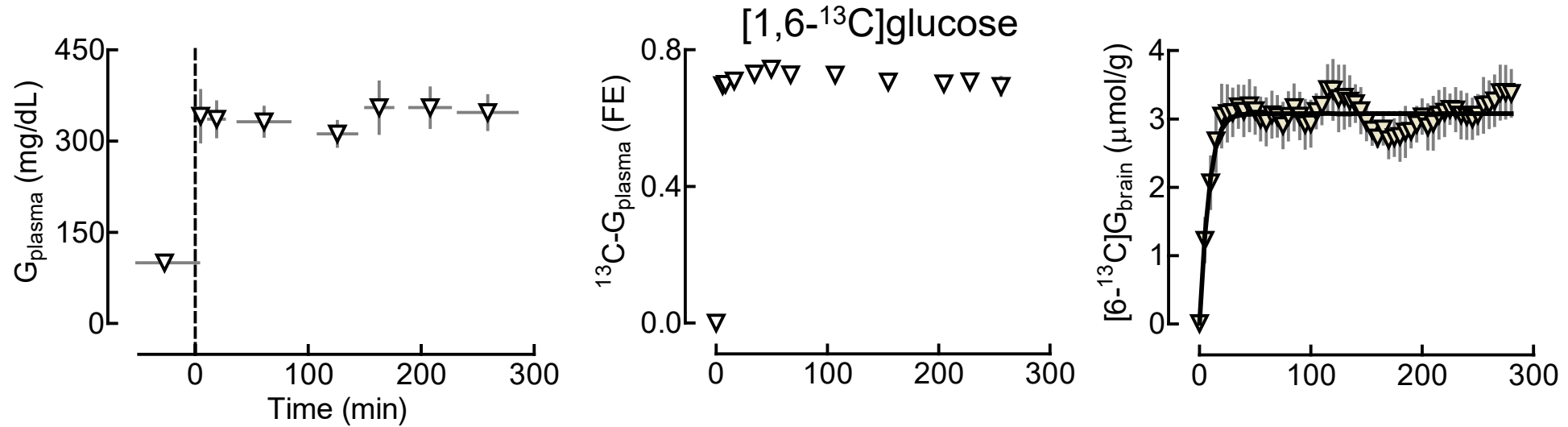
Large J-coupling constants between  $^{13}\text{C}$  and  $^1\text{H}$

$^1\text{H}$  decoupling to remove signal splitting

$^{13}\text{C}$  signals are spread over a much wider range than  $^1\text{H}$  signals



# Glucose transport by $^{13}\text{C}$ MRS – similar kinetic modelling approach



$$\frac{d^{13}G_{\text{brain}}}{dt} = T_{\text{max}} \frac{{}^{13}G_{\text{plasma}}(t) - \frac{{}^{13}G_{\text{brain}}(t)}{V_d}}{K_t + \frac{G_{\text{brain}}(t)}{V_d} + G_{\text{plasma}}(t)} - \text{CMR}_{\text{glc}} \frac{{}^{13}G_{\text{brain}}(t)}{G_{\text{brain}}(t)}$$

$$\frac{dG_{\text{brain}}}{dt} = T_{\text{max}} \frac{G_{\text{plasma}}(t) - \frac{G_{\text{brain}}(t)}{V_d}}{K_t + \frac{G_{\text{brain}}(t)}{V_d} + G_{\text{plasma}}(t)} - \text{CMR}_{\text{glc}}$$

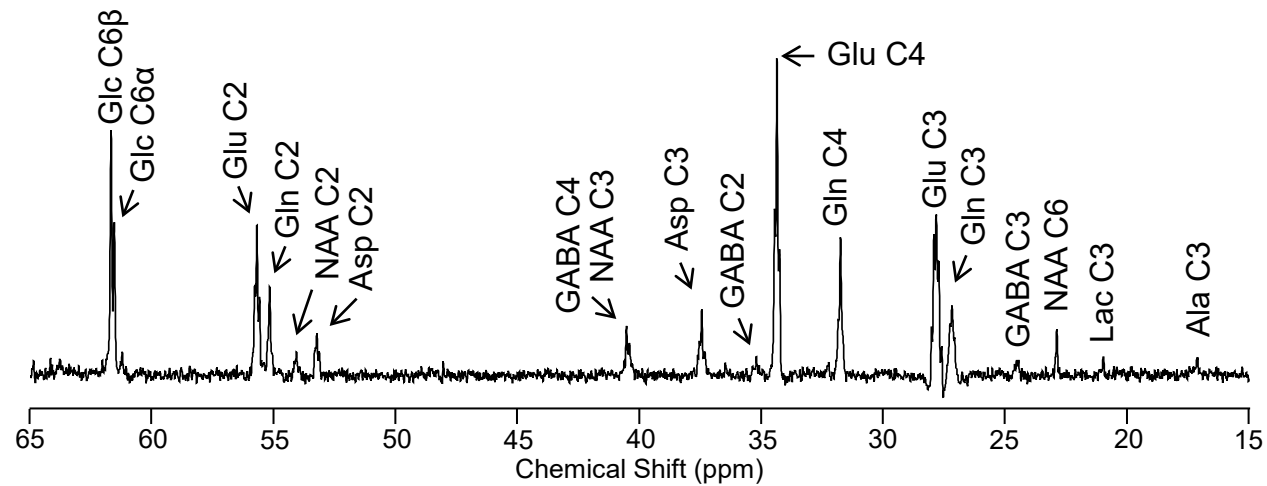
$$T_{\text{max}} = 0.91 \pm 0.03 \mu\text{mol/g/min}$$

$$K_t = 0.32 \pm 0.10 \text{ mM}$$

$$\text{CMR}_{\text{glc}} = 0.50 \pm 0.02 \mu\text{mol/g/min}$$

$$T_{\text{max}}/\text{CMR}_{\text{glc}} = 1.82 \pm 0.04$$

# $^{13}\text{C}$ -based metabolic flux analysis (MFA)



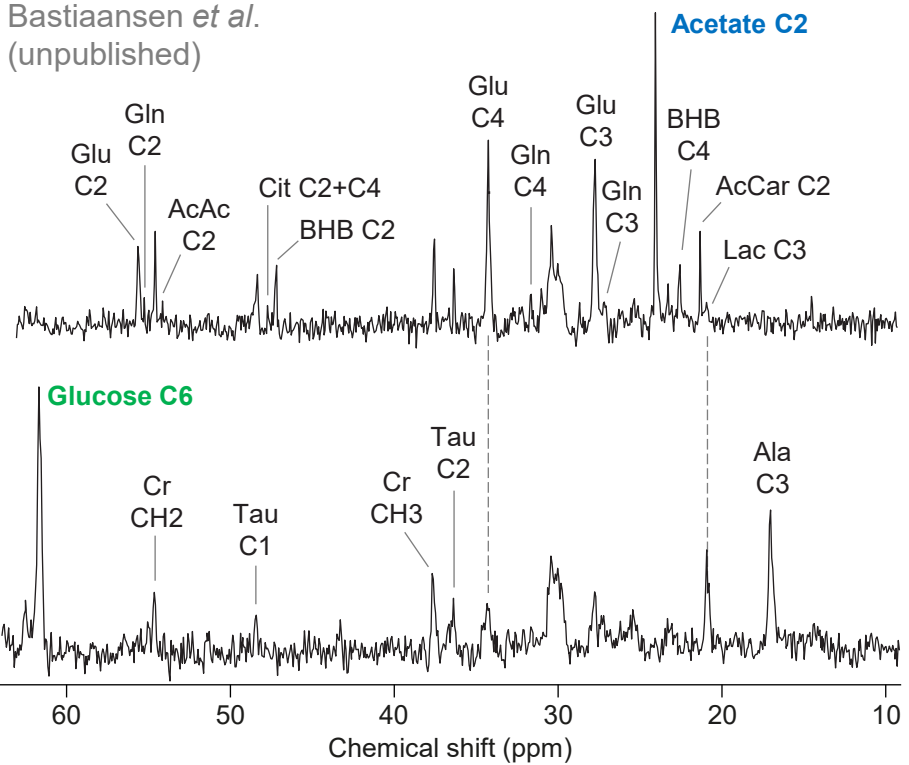
# Choice of the $^{13}\text{C}$ tracer

Must not affect the biochemical system

Dictates isotopomer distributions

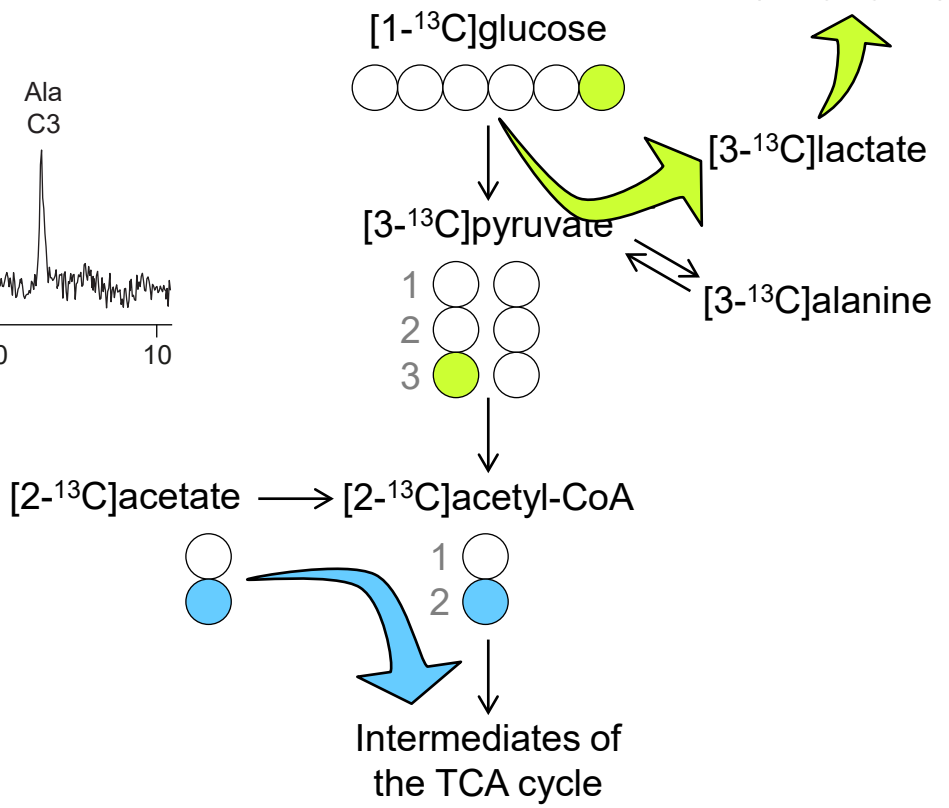
Influences flux estimation

Bastiaansen *et al.*  
(unpublished)

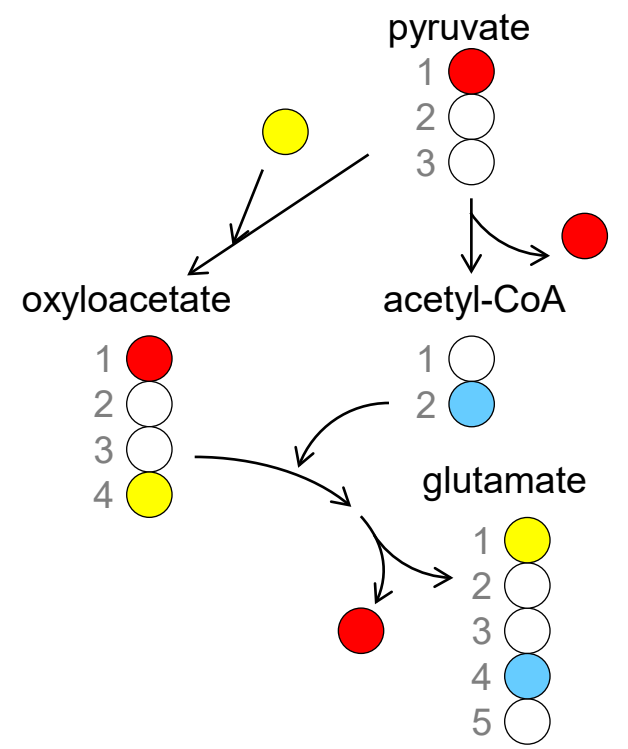


## Muscle

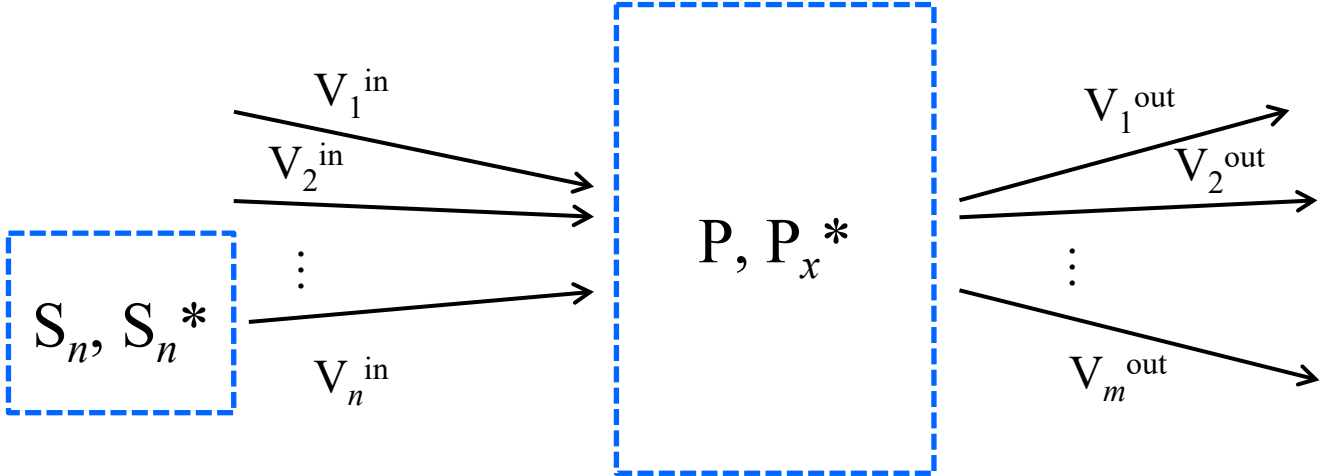
## Liver (Cory cycle)



## PC vs. PDH



# Modeling principles



**Balance equations:**

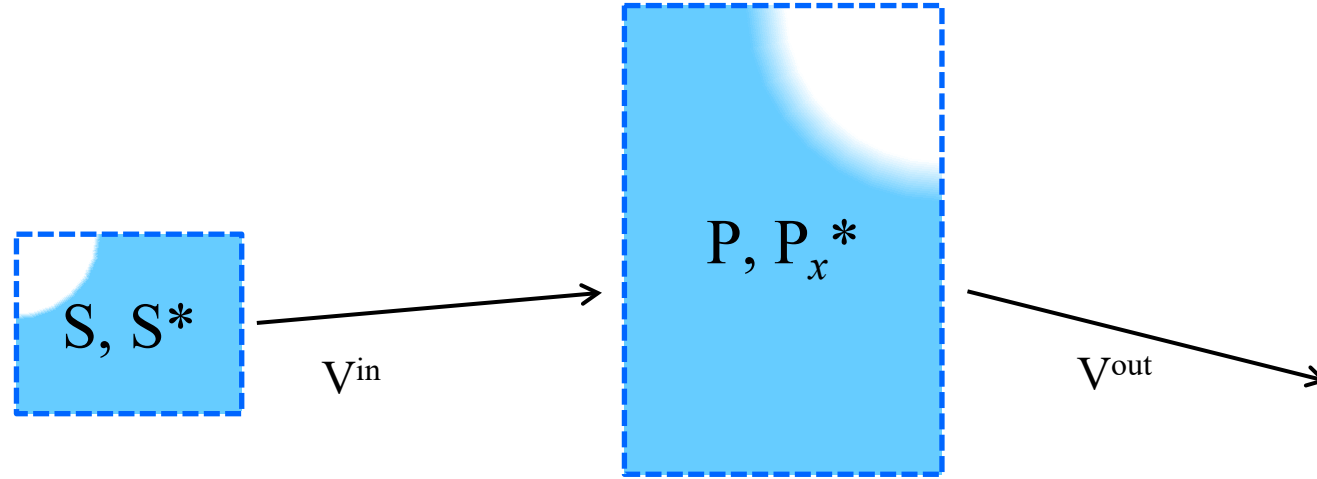
$$\frac{dP(t)}{dt} = \sum_i^n V_i^{in} - \sum_j^m V_j^{out} = 0$$

$$\frac{dP_x^*(t)}{dt} = \sum_i^n V_i^{in} \frac{S_i^*(t)}{S} - \sum_j^m V_j^{out} \frac{P_x^*(t)}{P}$$

P - whole content  
 P<sub>x</sub>\* - label fraction

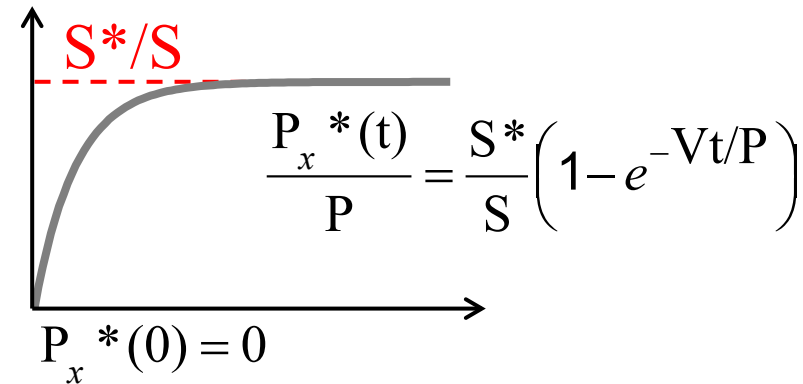
Fractional Enrichment  
 FE = P<sub>x</sub>\* / P

# Example: single pool, substrate with constant FE

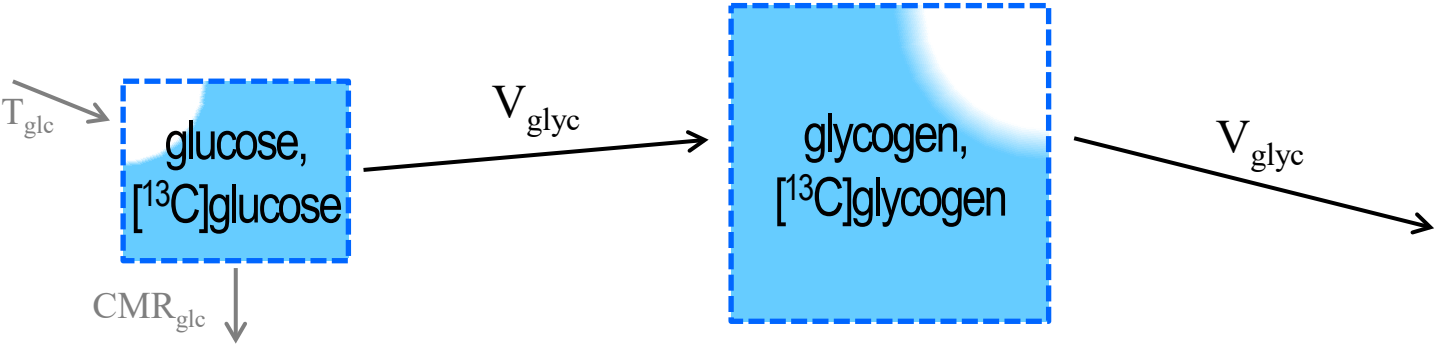


$$\frac{dP(t)}{dt} = V^{in} - V^{out} = 0$$

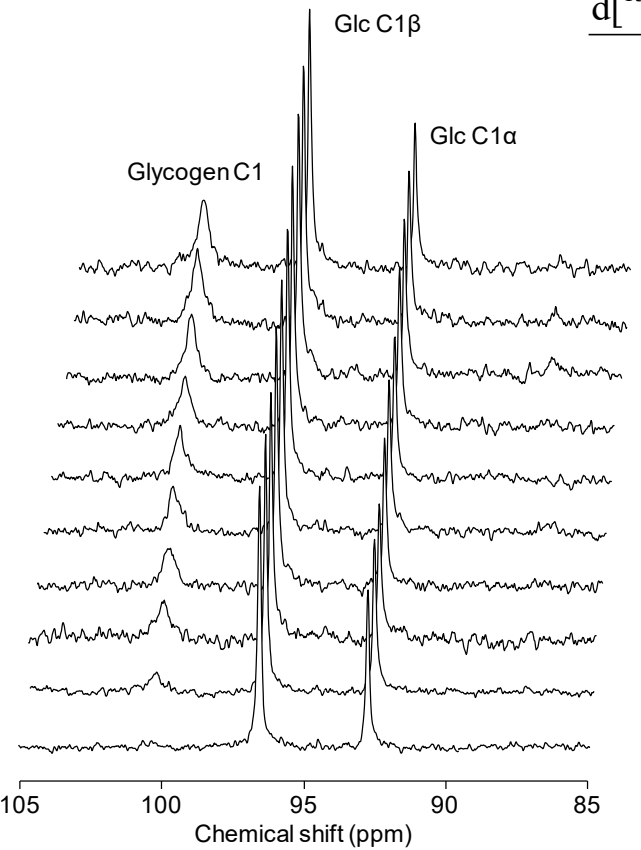
$$\frac{dP_x^*(t)}{dt} = V^{in} \frac{S^*(t)}{S} - V^{out} \frac{P_x^*(t)}{P}$$



# Brain glycogen metabolism in the diabetic brain

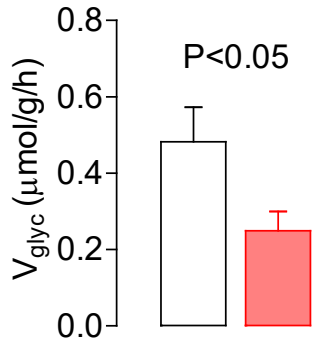


$$\frac{d[^{13}\text{C}]\text{glycogen}}{dt} = V_{\text{glyc}} \left( \frac{[^{13}\text{C}]\text{glucose}}{\text{glucose}} - \frac{[^{13}\text{C}]\text{glycogen}}{\text{glycogen}} \right)$$

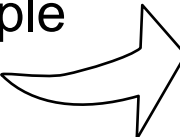


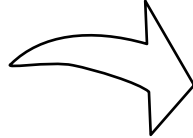
*important assumptions:*

- (1) labelling of glucose-6-phosphate resembles that of glucose because  $V_{\text{glyc}} \ll \text{CMR}_{\text{glc}}$
- (2) glycogen concentration is constant, implying that synthesis and degradation fluxes are equal ( $V_{\text{glyc}}$ )



# Main ingredients for designing MFA models

! **Abstraction level:** models should be as simple as possible and as complex as necessary.  focus on pathways of interest vs. neglect presence or influence of all other processes

! **Modelling principles:**  
• Mass and energy conservation.  
Fluxes represented in terms of the system state (metabolite concentrations)  Set of balance equations describing system dynamics and/or equilibrium

! **Assumptions** and data sources must be documented precisely.

Judge applicability conditions!

# Construction of a metabolic model for $^{13}\text{C}$ MFA

## 1 primary assumptions

Enzymes do not discriminate between  $^{13}\text{C}$  and  $^{12}\text{C}$

not true for  $\text{CO}_2$ -fixing enzyme Rubisco  
*Plant Physiol* 101:37, 1993

Each metabolite pool is homogeneous

There's no diffusion limitation within the same compartment

not true for channelling of metabolites in multi-enzyme complexes  
*Trends Biochem Sci* 15:411, 1990

Labelling in a metabolite pool is well mixed before each reaction



Probability of leaving the pool is the same for all molecules

# Construction of a metabolic model for $^{13}\text{C}$ MFA

## 2 stoichiometric balance equations

### Model reduction/simplification

Omit:

- ✓ repressed pathways
- ✓ cofactors

Combine:

- ✓ isoenzymes
- ✓ linear and non-bifurcated pathways
- ✓ intermediate pools that equilibrate rapidly

### Keep complexity to an extent that is feasible

Reversibility  
of reactions

Compartmentation:  
Location/site of reactions;  
transport across compartments

Reactions with  $\text{CO}_2$  fixation:  
uncertain dilution due to  
decarboxylation reactions

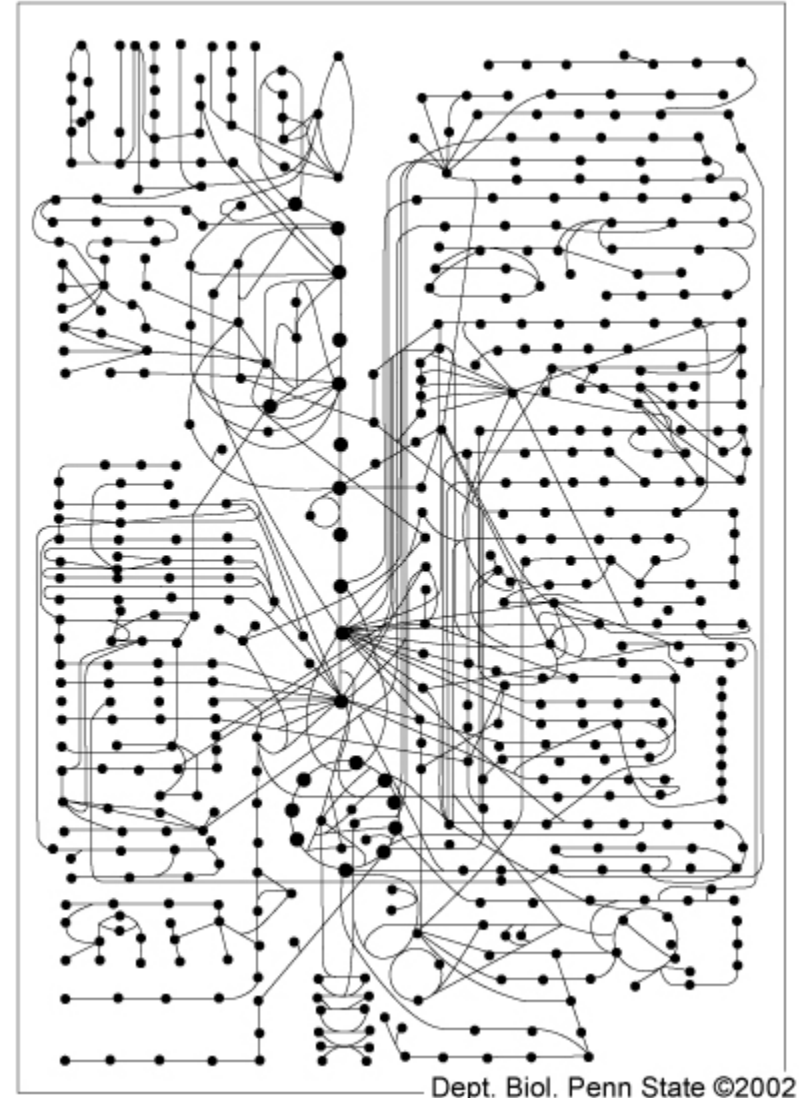


Diagram depicting only a fraction of the metabolic pathways in a cell.

# Construction of a metabolic model for $^{13}\text{C}$ MFA

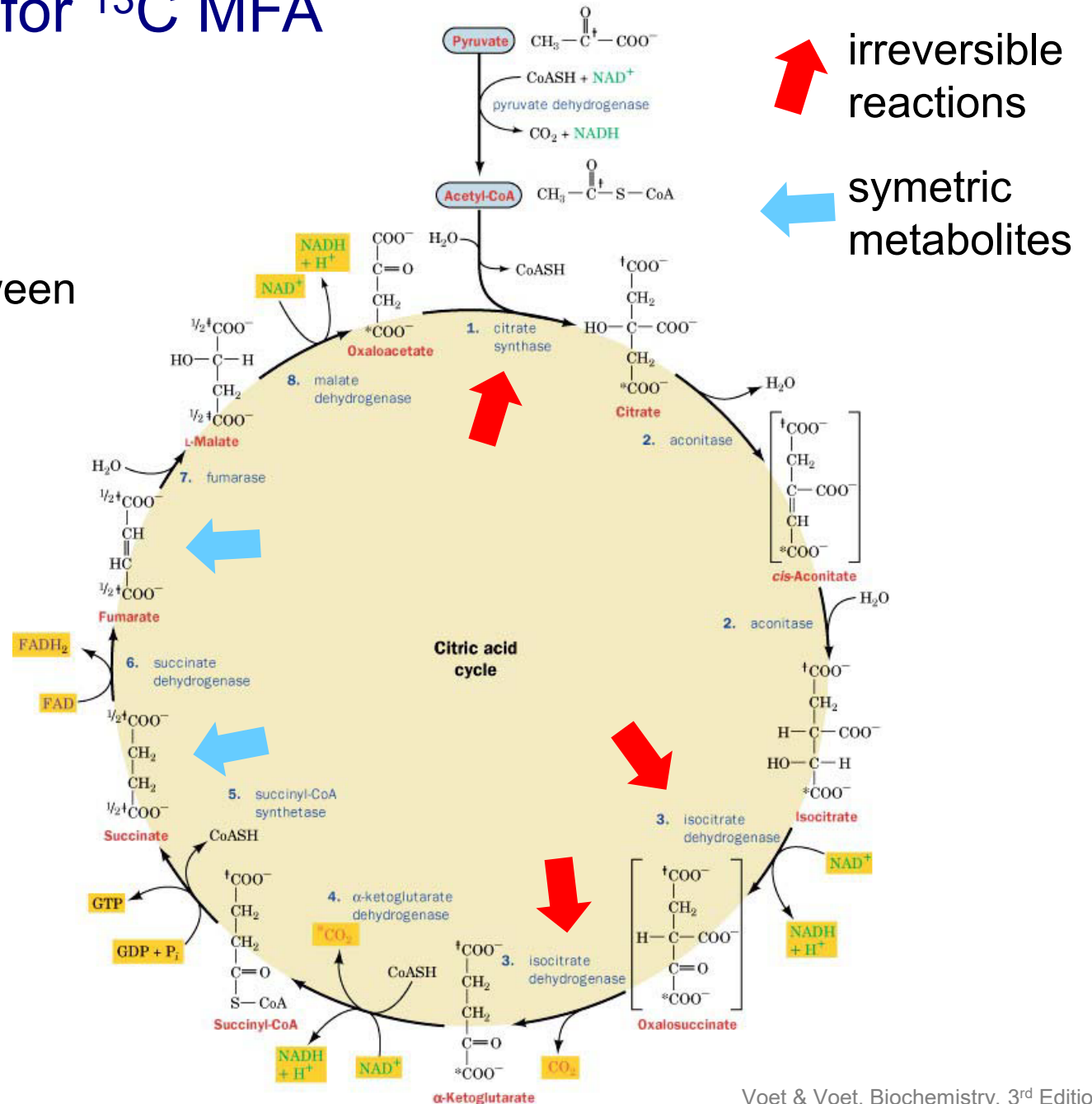
## 3 mapping carbon atoms

Describe matching of carbon atoms between each metabolite and its precursor.

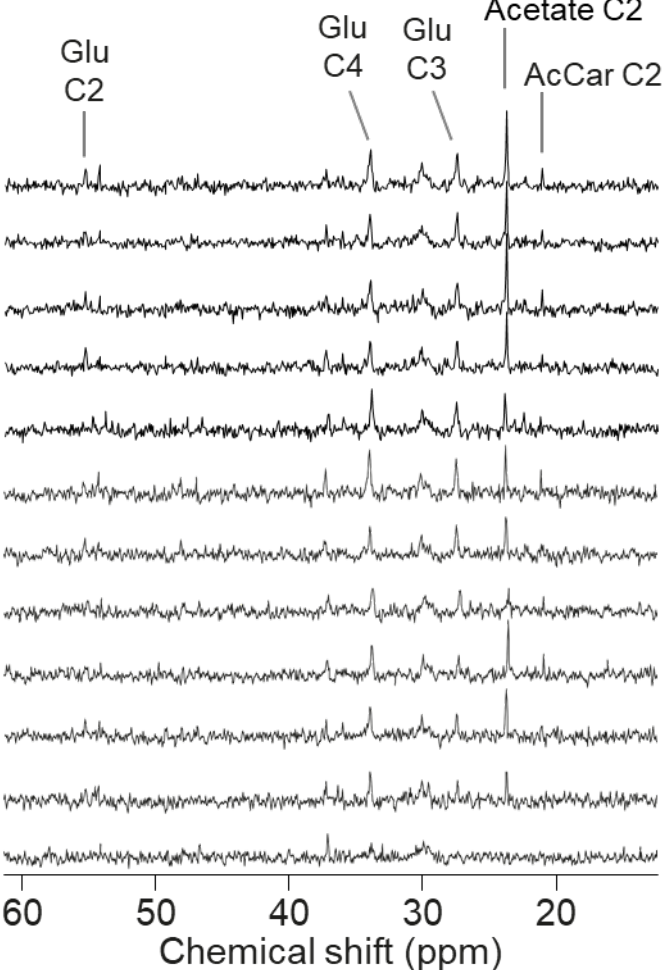
- ✓ Mostly conserved among species
- ✓ Available from pre-existing models

Center of symmetry in molecules causes labelling redistribution.

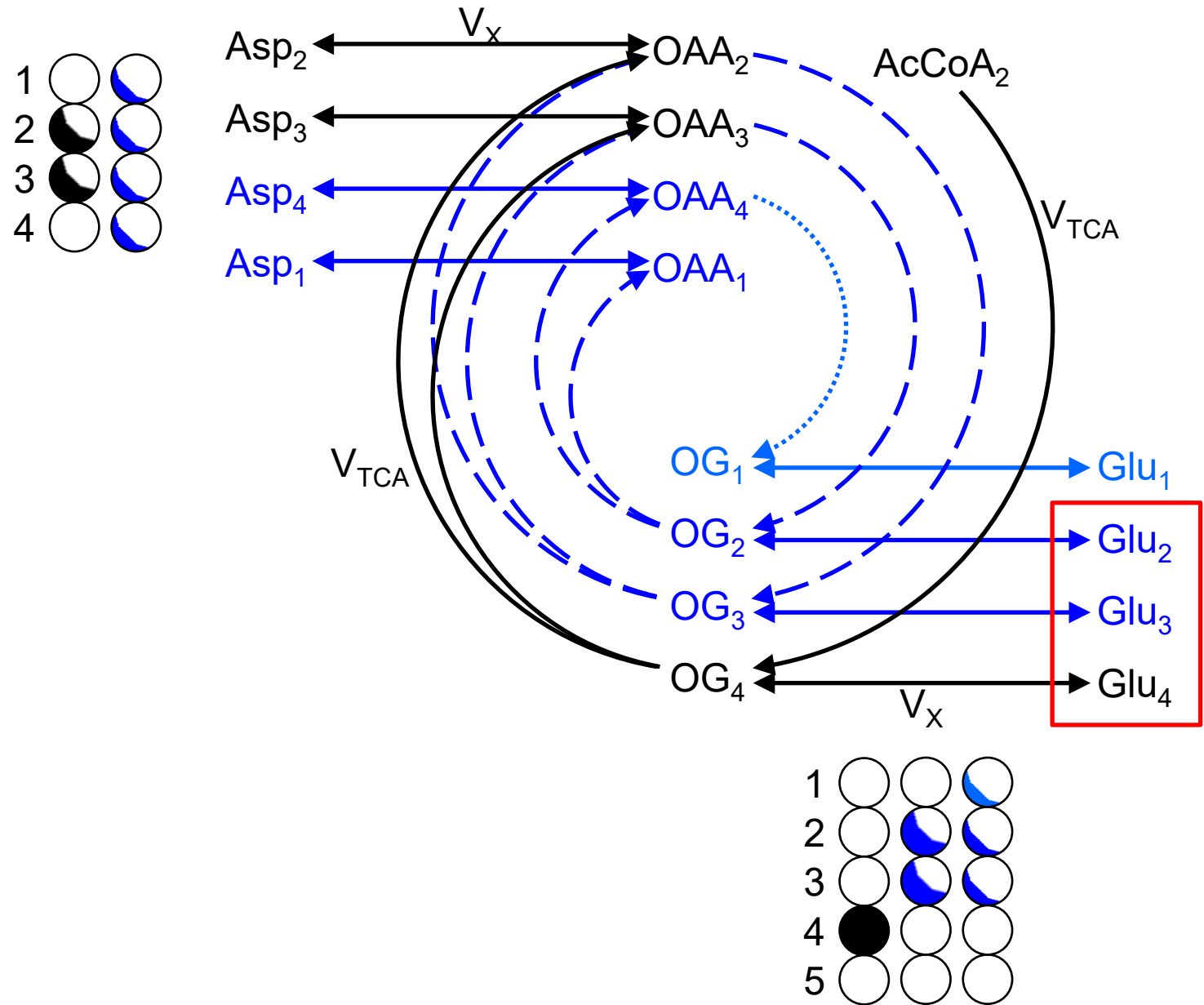
Origin of  $^{13}\text{C}$  labelling, dilution reactions and labelling loss



# TCA cycle flux in the skeletal muscle



# $^{13}\text{C}$ labeling of TCA cycle intermediates

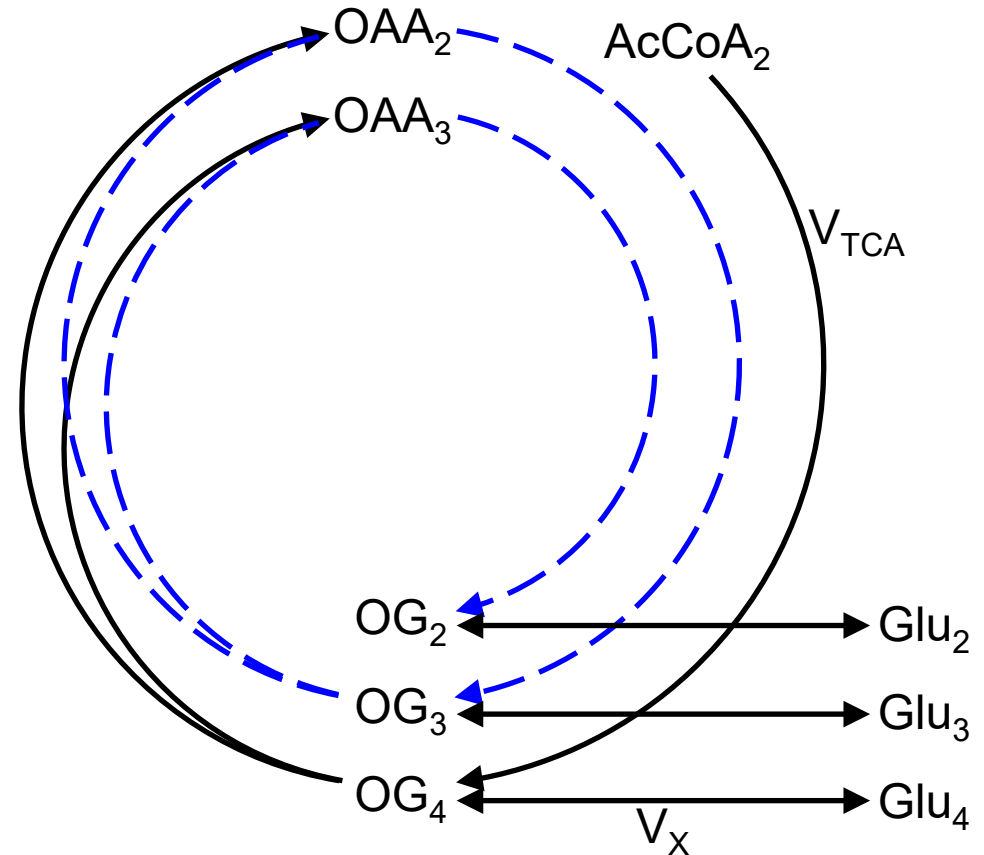


# $^{13}\text{C}$ labeling of TCA cycle intermediates

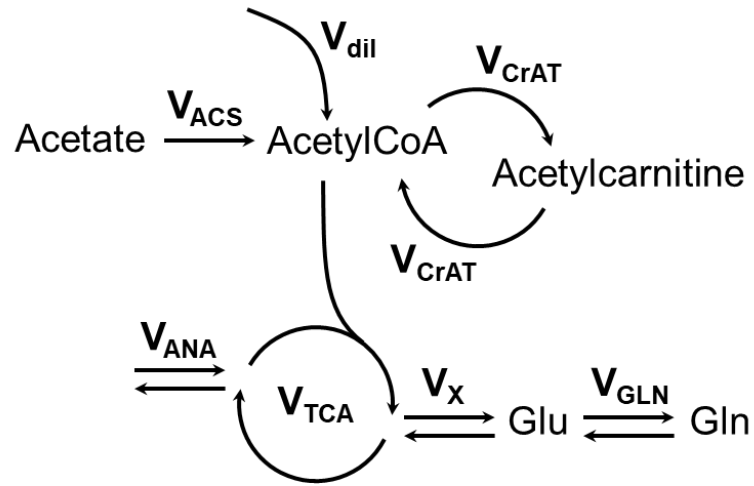
Mathematical equations defining isotopic labelling

$$\begin{cases} \frac{d^{13}\text{OG}_4}{dt} = V_{\text{TCA}} \frac{^{13}\text{AcCoA}_2}{\text{AcCoA}} + V_X \frac{^{13}\text{Glu}_4}{\text{Glu}} - (V_{\text{TCA}} + V_X) \frac{^{13}\text{OG}_4}{\text{OG}} \\ \frac{d^{13}\text{OG}_3}{dt} = V_{\text{TCA}} \frac{^{13}\text{OAA}_2}{\text{OAA}} + V_X \frac{^{13}\text{Glu}_3}{\text{Glu}} - (V_{\text{TCA}} + V_X) \frac{^{13}\text{OG}_3}{\text{OG}} \\ \frac{d^{13}\text{OG}_2}{dt} = V_{\text{TCA}} \frac{^{13}\text{OAA}_3}{\text{OAA}} + V_X \frac{^{13}\text{Glu}_2}{\text{Glu}} - (V_{\text{TCA}} + V_X) \frac{^{13}\text{OG}_2}{\text{OG}} \end{cases}$$

$$\begin{cases} \frac{d^{13}\text{OAA}_2}{dt} = \frac{1}{2} V_{\text{TCA}} \frac{^{13}\text{OG}_4 + ^{13}\text{OG}_3}{\text{OG}} + V_{\text{ANA}} 0.011 - (V_{\text{TCA}} + V_{\text{ANA}}) \frac{^{13}\text{OAA}_2}{\text{OAA}} \\ \frac{d^{13}\text{OAA}_3}{dt} = \frac{1}{2} V_{\text{TCA}} \frac{^{13}\text{OG}_4 + ^{13}\text{OG}_3}{\text{OG}} + V_{\text{ANA}} 0.011 - (V_{\text{TCA}} + V_{\text{ANA}}) \frac{^{13}\text{OAA}_3}{\text{OAA}} \end{cases}$$



# Designing the mathematical model



## Mathematical equations defining isotopic labelling

$$\frac{d^{13}\text{AcCoA}_2}{dt} = V_{AC} \frac{^{13}\text{Ac}_2(t)}{\text{Ac}(t)} + V_{dil} 0.011 - V_{TCA} \frac{^{13}\text{AcCoA}_2}{\text{AcCoA}}$$

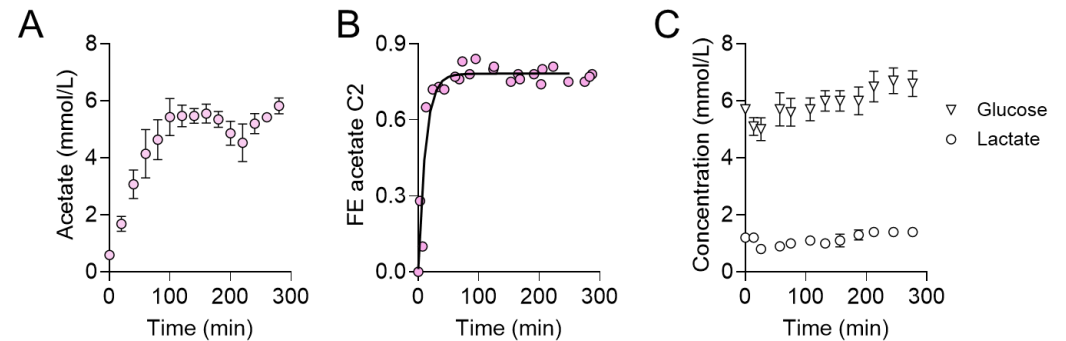
$$\frac{d^{13}\text{Glu}_i}{dt} = V_X \left( \frac{^{13}\text{OG}_2}{\text{OG}} - \frac{^{13}\text{Glu}_i}{\text{Glu}} \right)$$

## Mass balance equations at steady-state

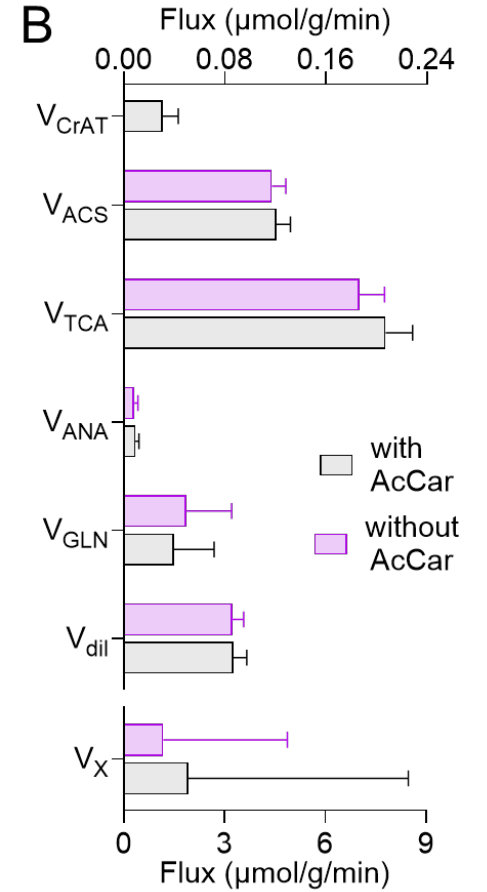
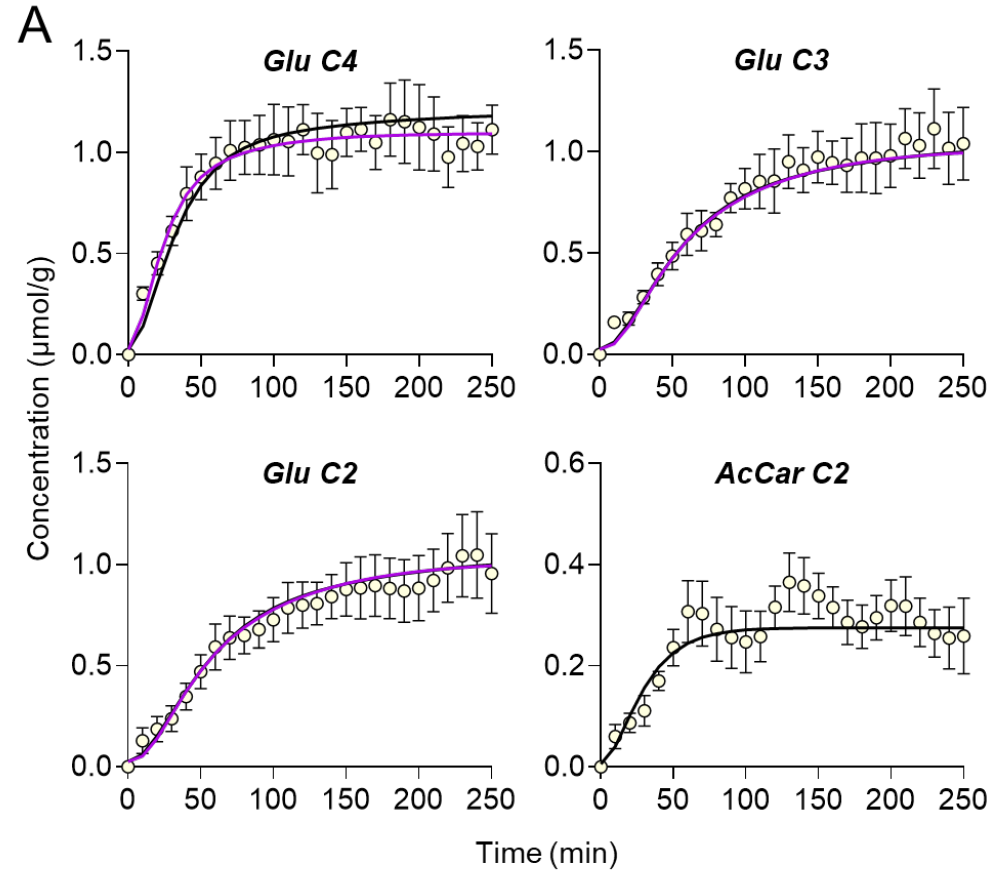
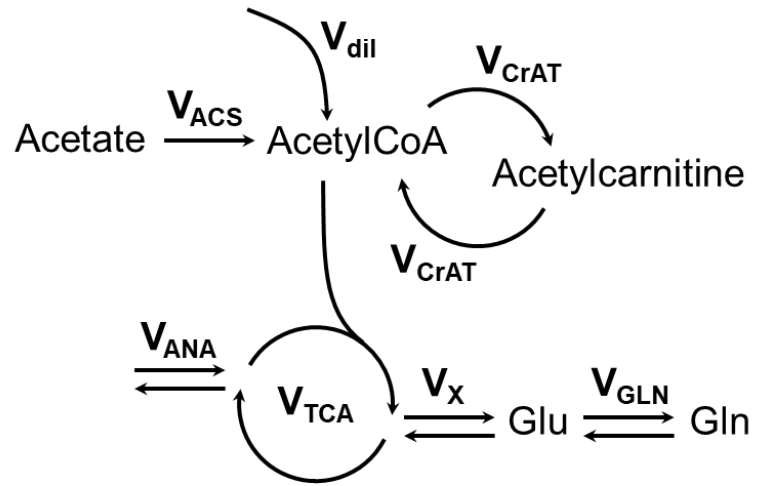
$$\frac{d\text{OAA}}{dt} = \frac{d\text{OG}}{dt} = \frac{d\text{Glu}}{dt} = 0$$

$$\frac{d\text{AcCoA}}{dt} = V_{AC} + V_{dil} - V_{TCA} = 0 \Rightarrow V_{TCA} = V_{AC} + V_{dil}$$

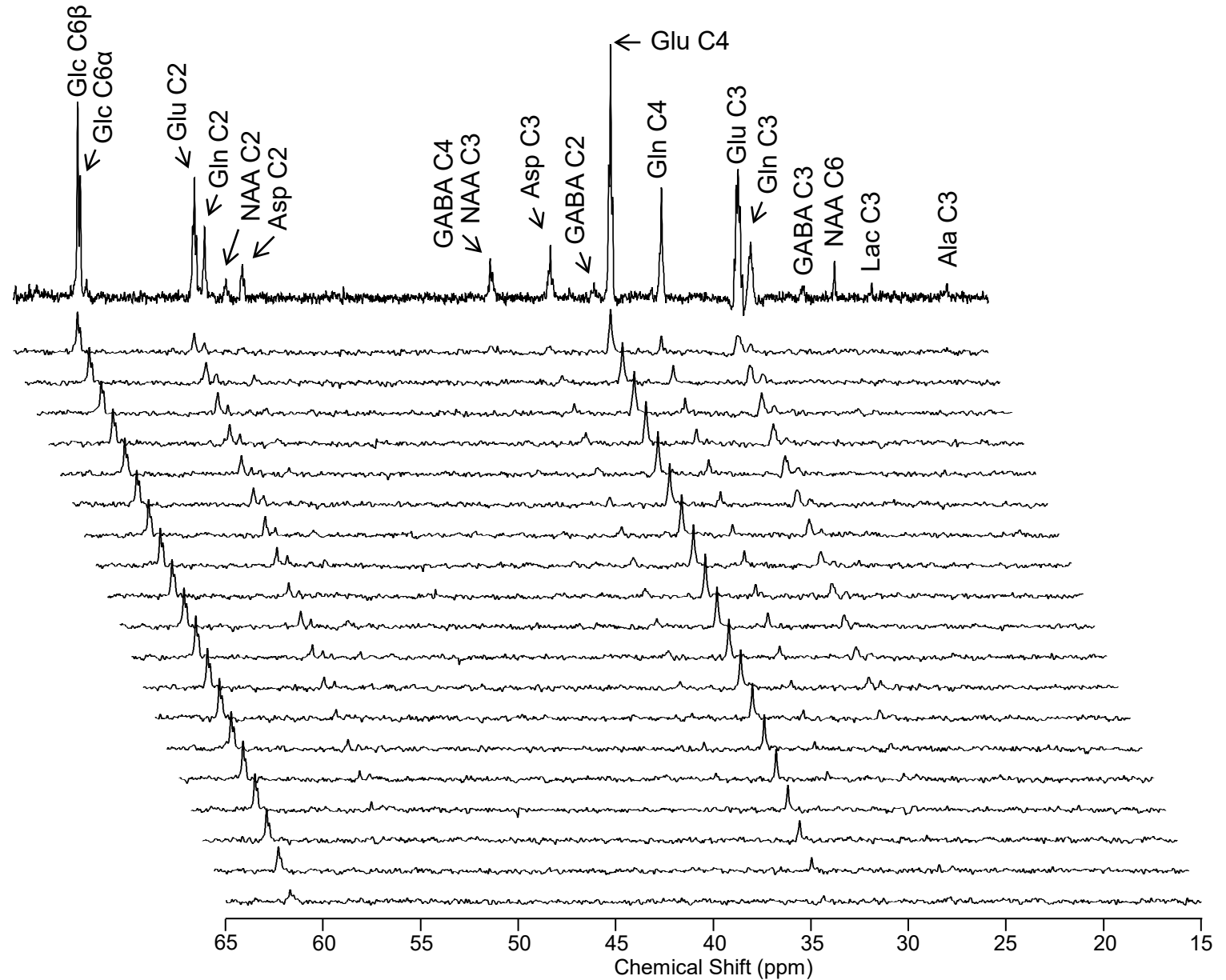
## “input function” from plasma [2-<sup>13</sup>C]acetate



# Designing the mathematical model



# $^{13}\text{C}$ MRS in the rat brain



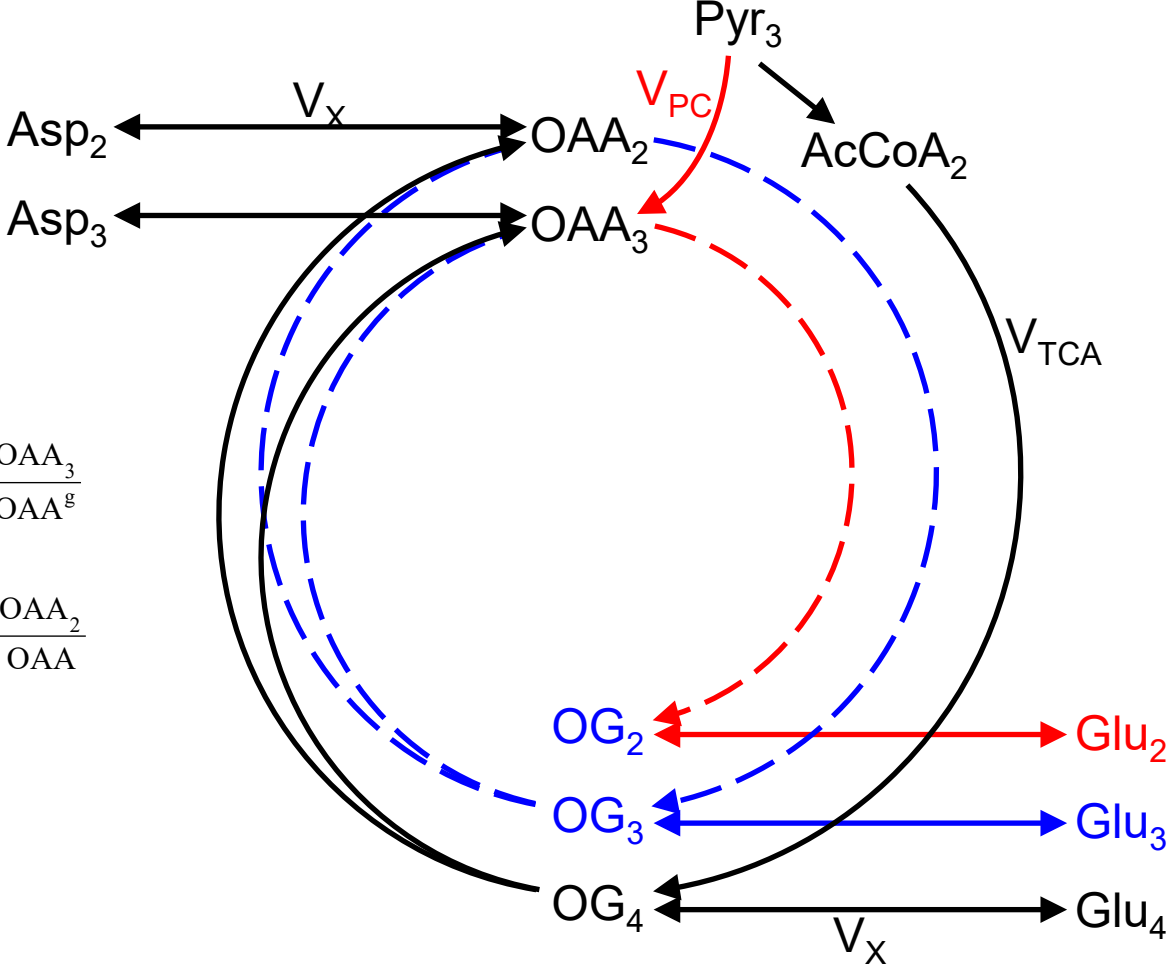
❖ Infusion of [1,6- $^{13}\text{C}$ ]glucose

❖ Direct detection by  $^{13}\text{C}$  MRS at 14.1 T

# Pyruvate carboxylation ( $V_{PC}$ )

$$\frac{dOAA_3^g}{dt} = \frac{V_g}{2} \left( \frac{OG_4 + OG_3}{OG^g} \right) + V_{PC} \frac{Pyr_3}{Pyr} + V_X \frac{Asp_3}{Asp^g} - (V_{TCA} + V_X) \frac{OAA_3}{OAA^g}$$

$$\frac{dOAA_2^g}{dt} = \frac{V_g}{2} \left( \frac{OG_4 + OG_3}{OG^g} \right) + V_{PC} 0.011 + V_X \frac{Asp_2}{Asp^g} - (V_{TCA} + V_X) \frac{OAA_2}{OAA}$$

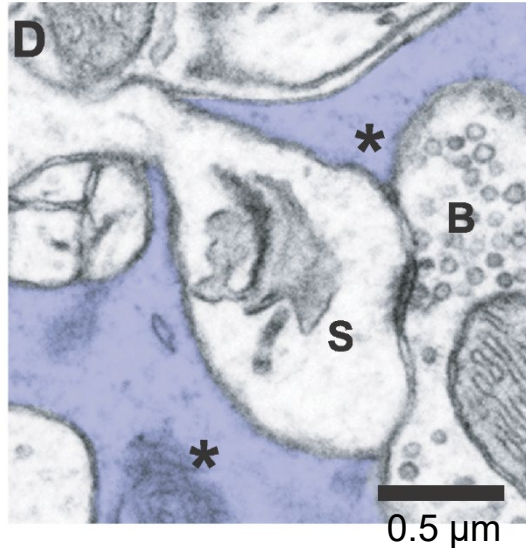


- 1 ○
- 2 ●
- 3 ○
- 4 ●
- 5 ○

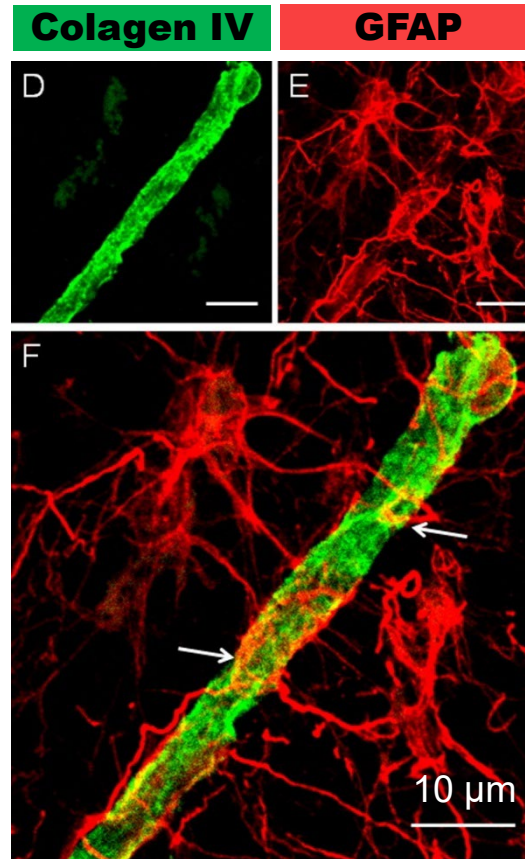
# The metabolic importance of astrocytes

Astrocytic endfeet cover blood vessels

Astrocytic processes engulfing synapses



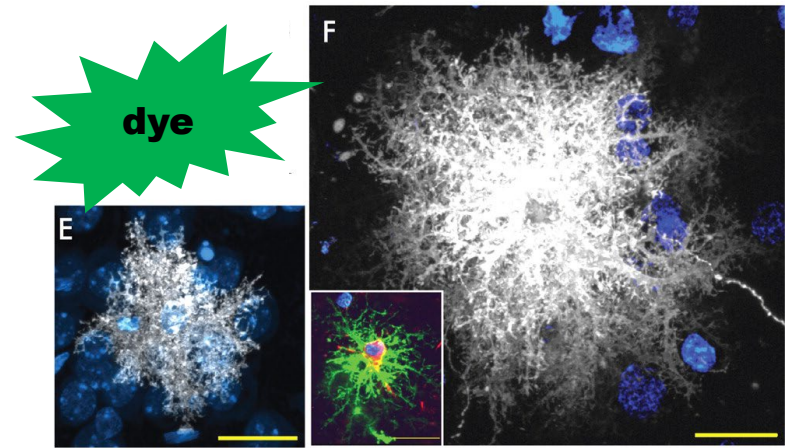
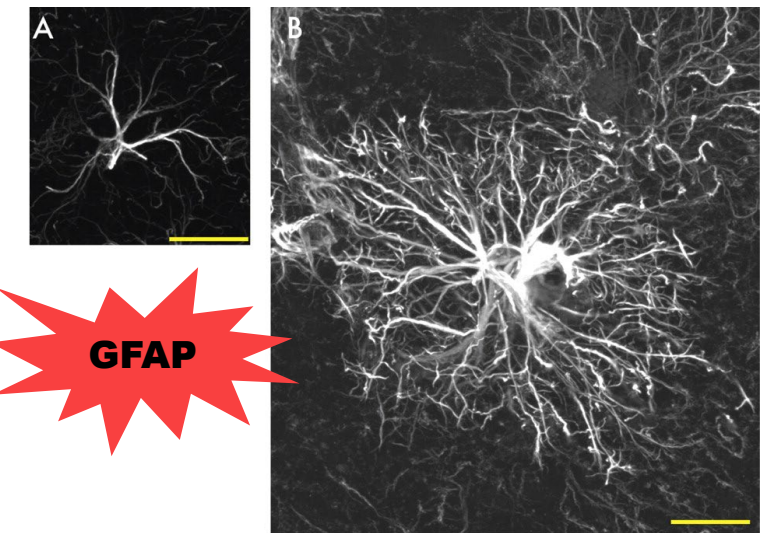
Genoud *et al.*, J Neurosci 2006



Rajkowska *et al.*, Biol Psy 2013

mouse astrocyte

human astrocyte

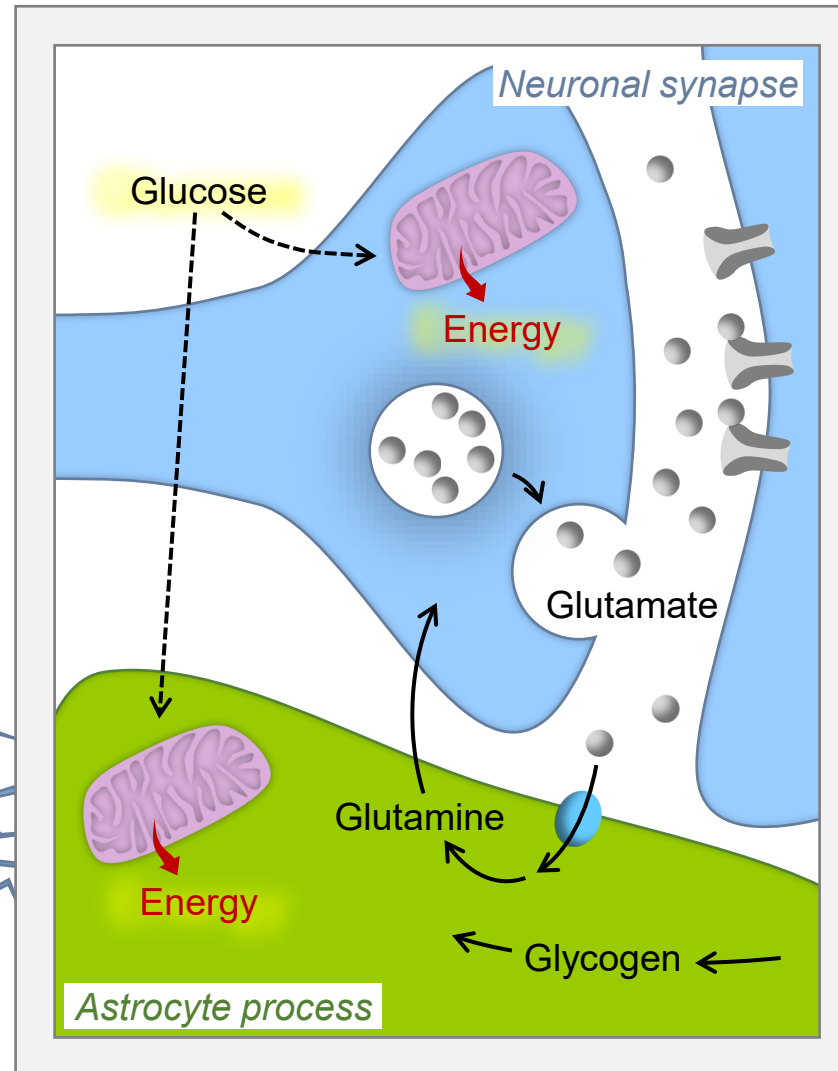
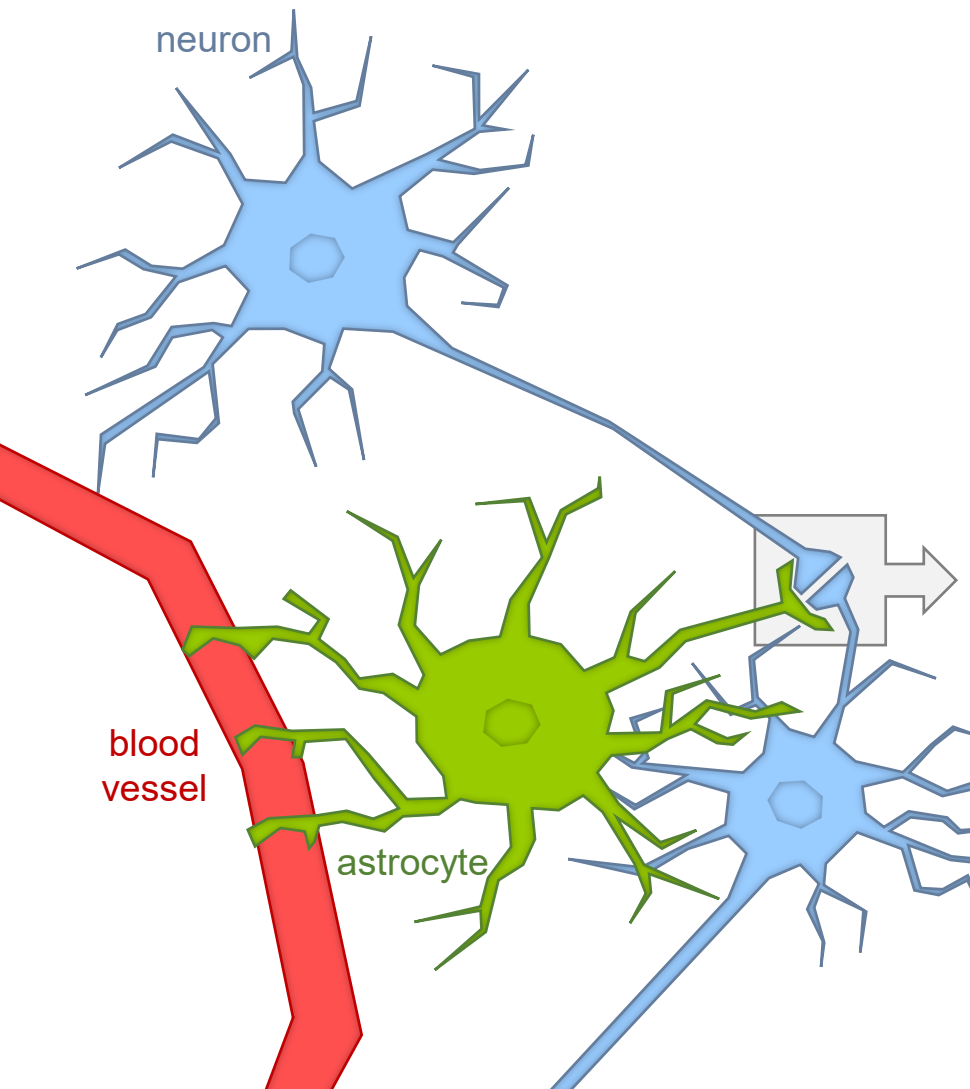


Oberheim *et al.*, J Neurosci 2009

20 μm

Further reading: NEWS FEATURE The 'silent' brain cells that shape our behaviour, memory and health. Nature 648, 23-25 (2025)

# Astrocytes participate in the glutamate-glutamine cycle

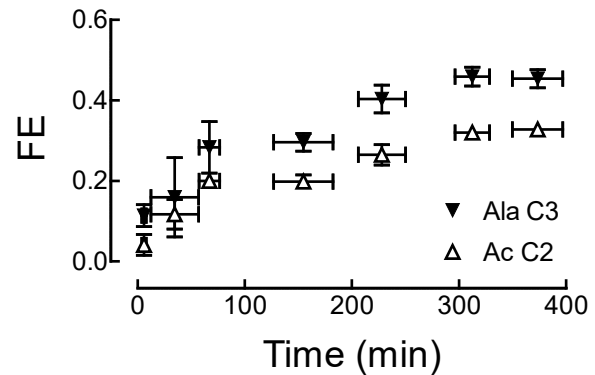
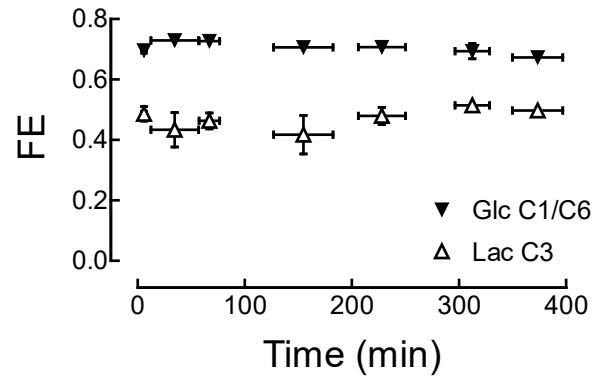


## Brain glutamate

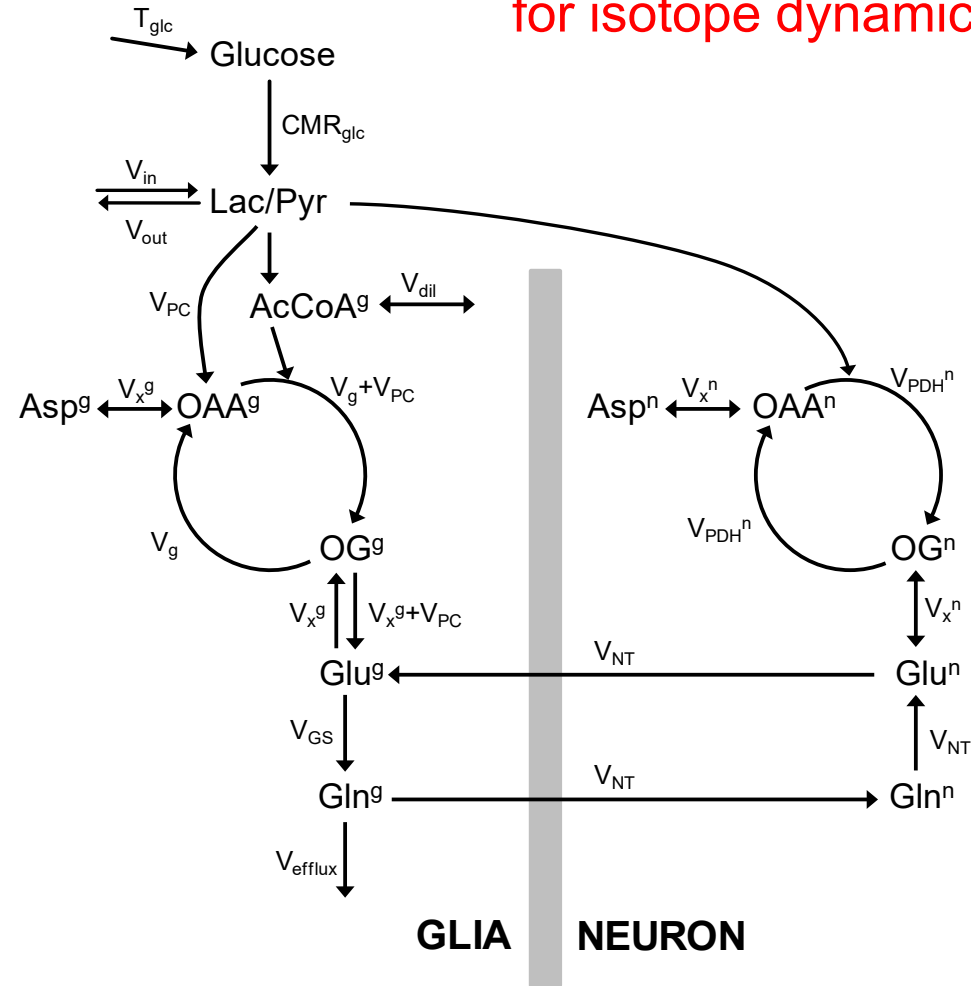
- Excitatory neurotransmitter
- Key role in plasticity and memory performance
- The most abundant free amino acid
- At the crossroad between multiple metabolic pathways
- Synthesized *de novo* from glucose

# Model of brain glucose metabolism

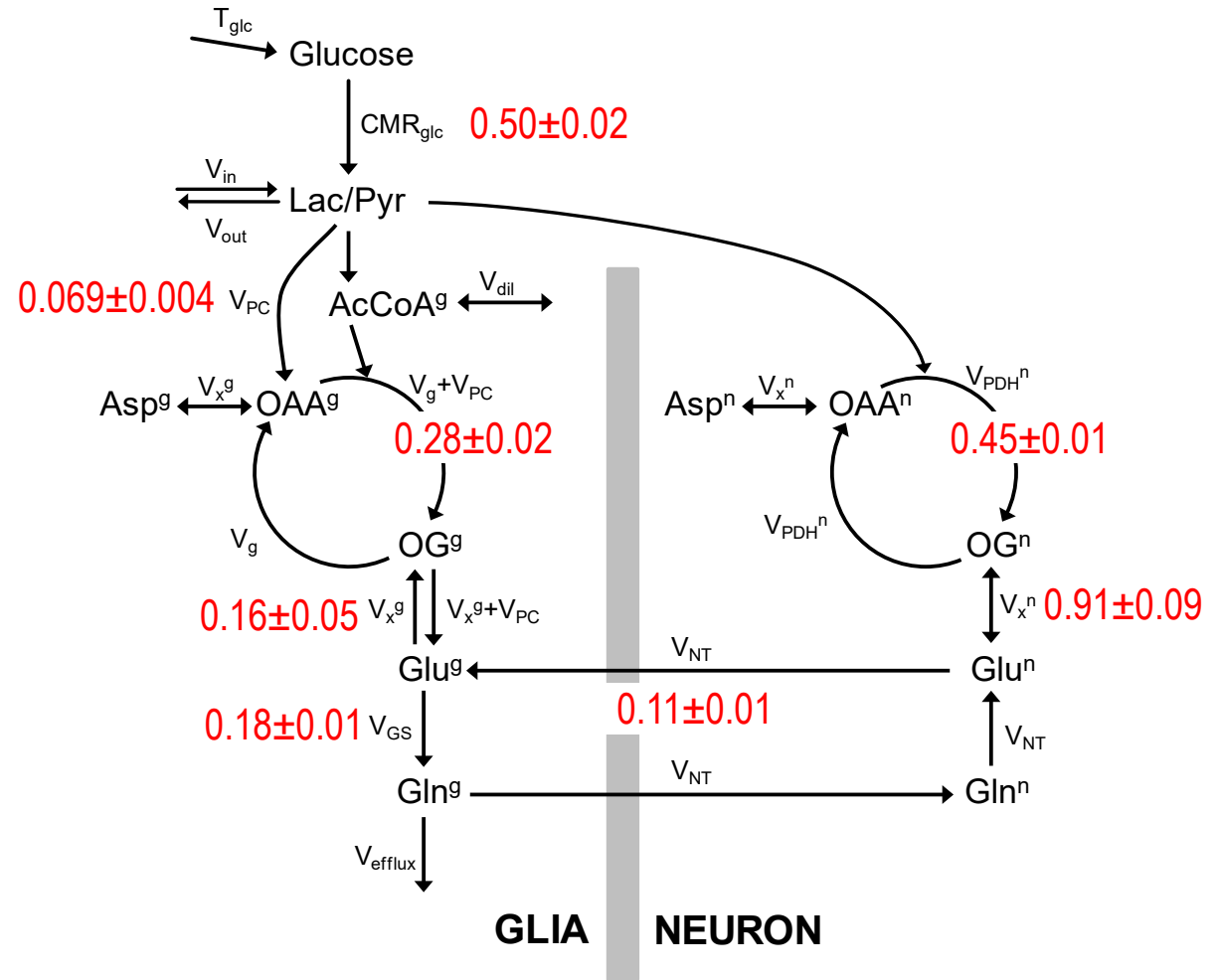
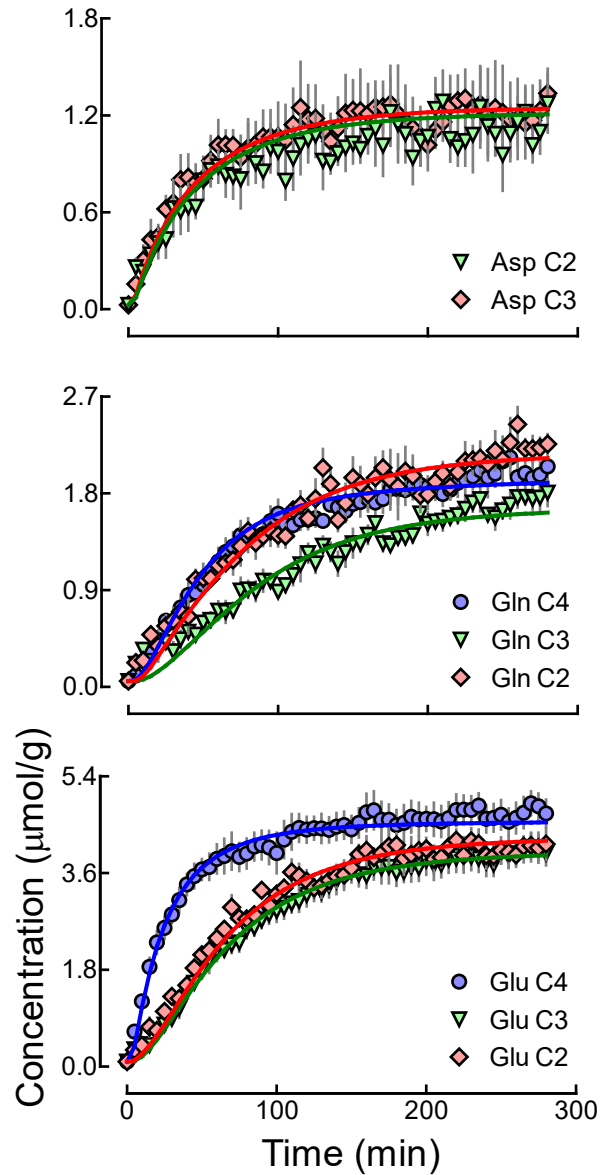
“input function” from plasma metabolites



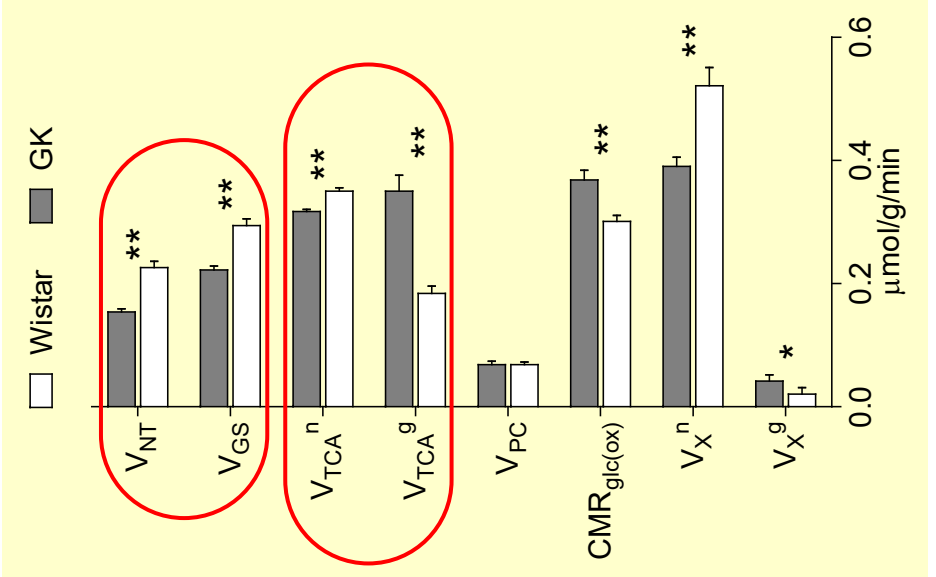
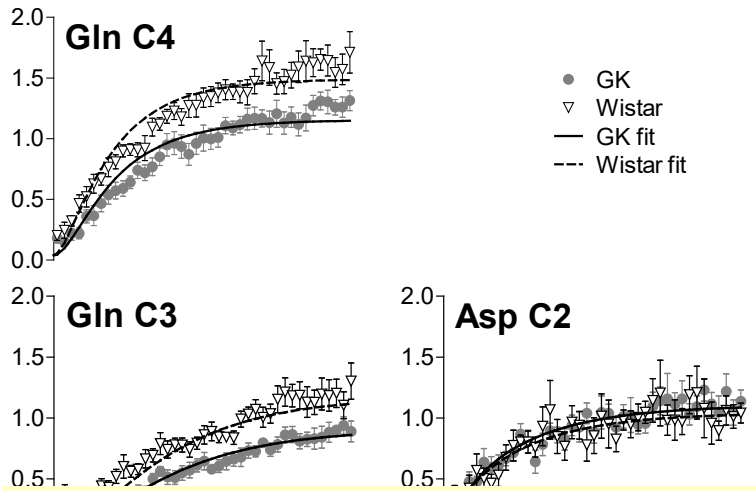
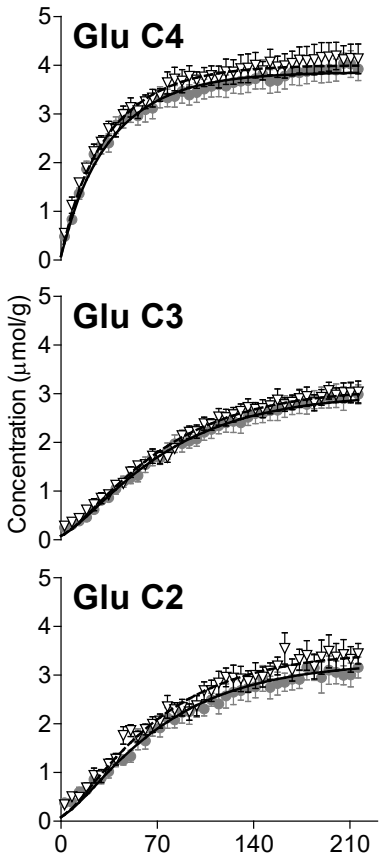
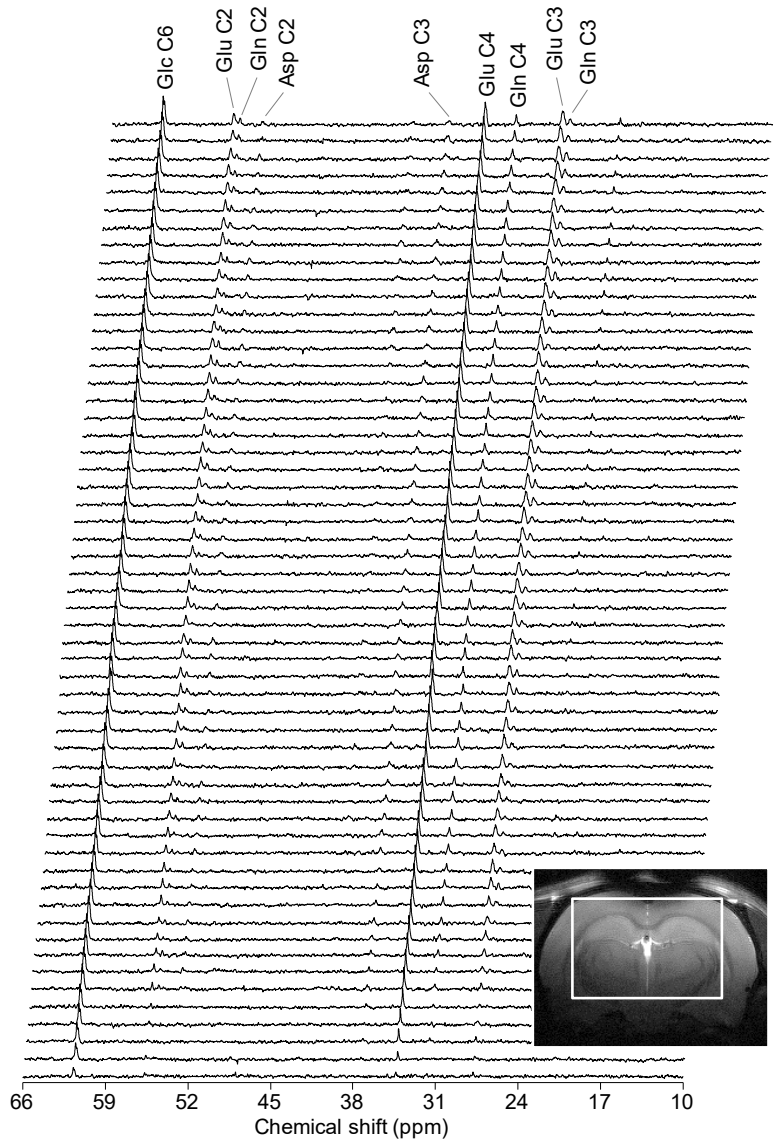
35 differential equations for isotope dynamics



# $^{13}\text{C}$ -enriched metabolites in the brain

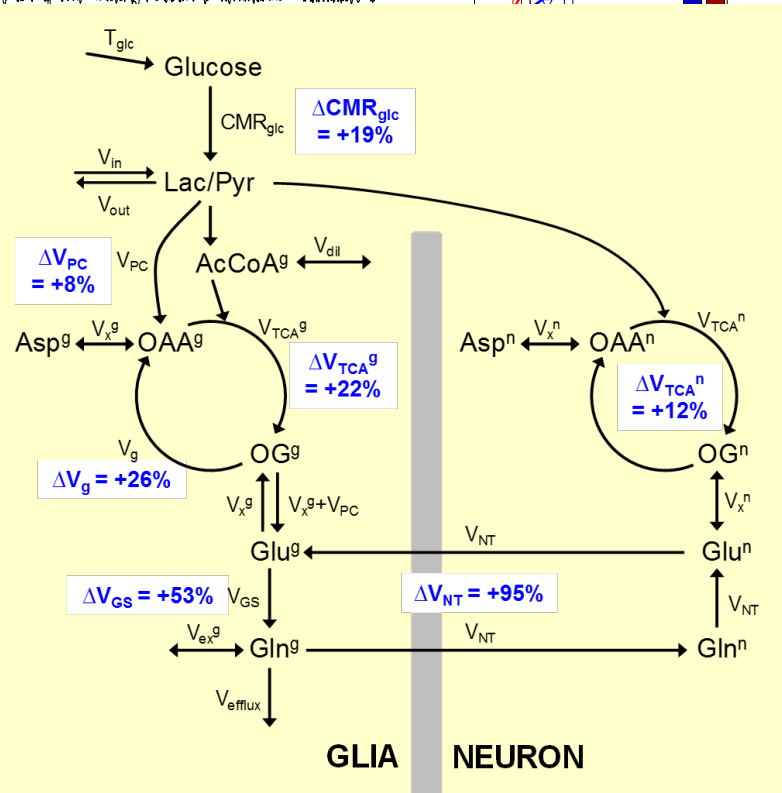
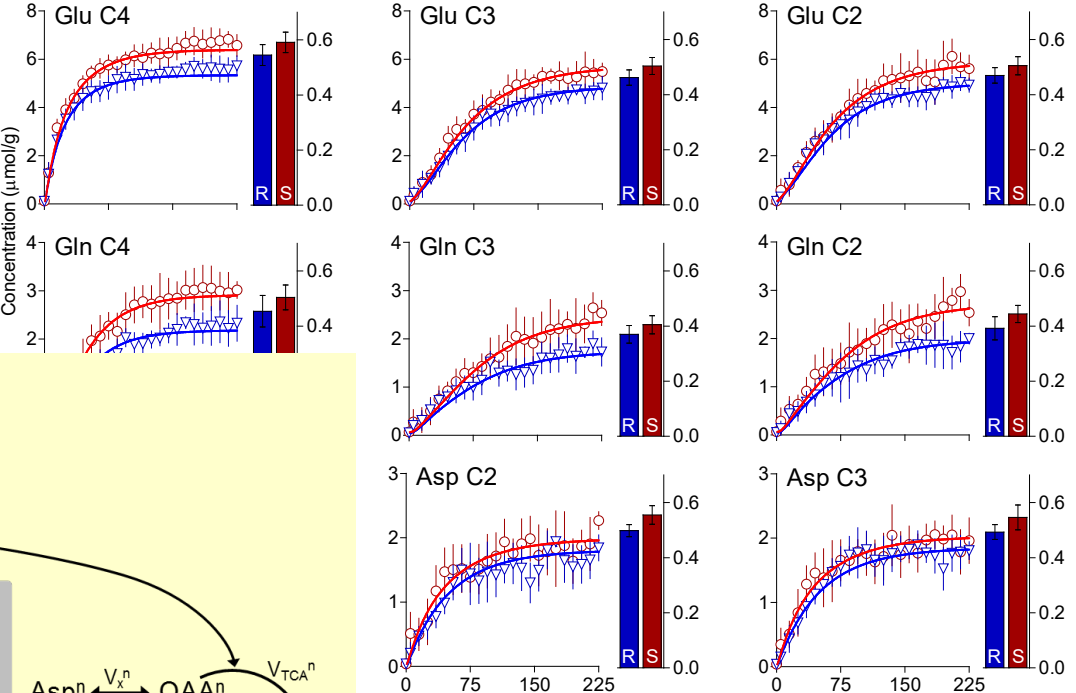
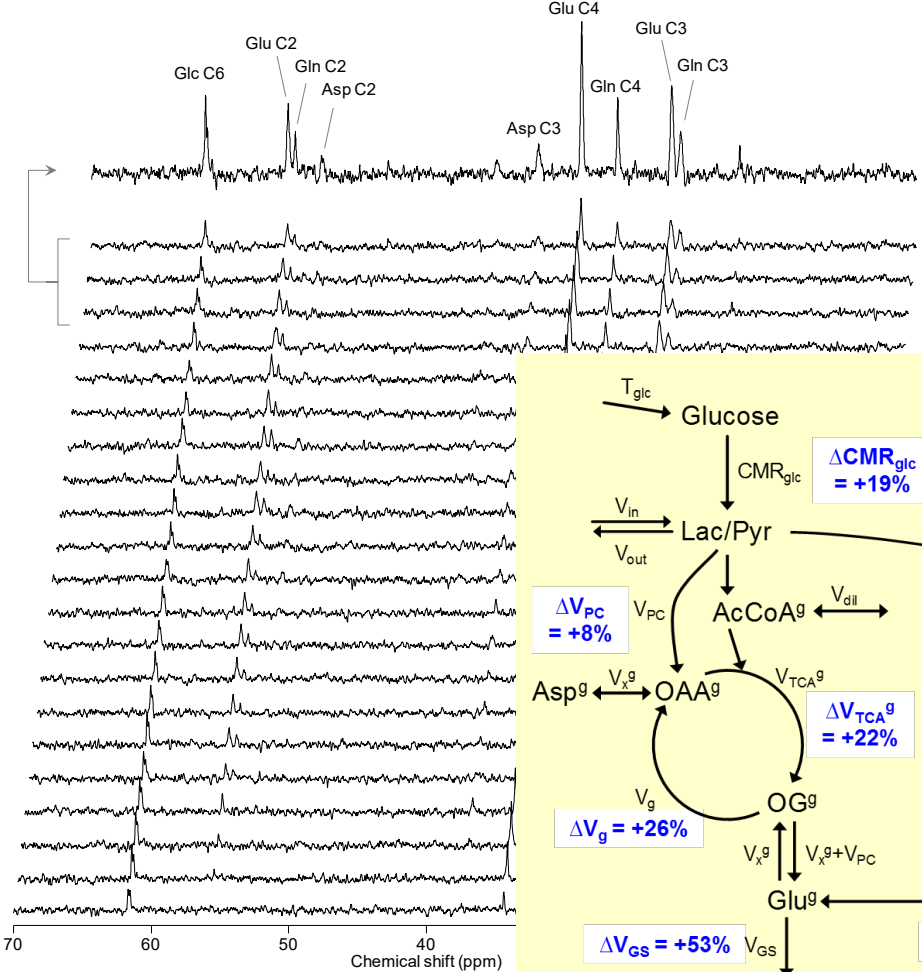


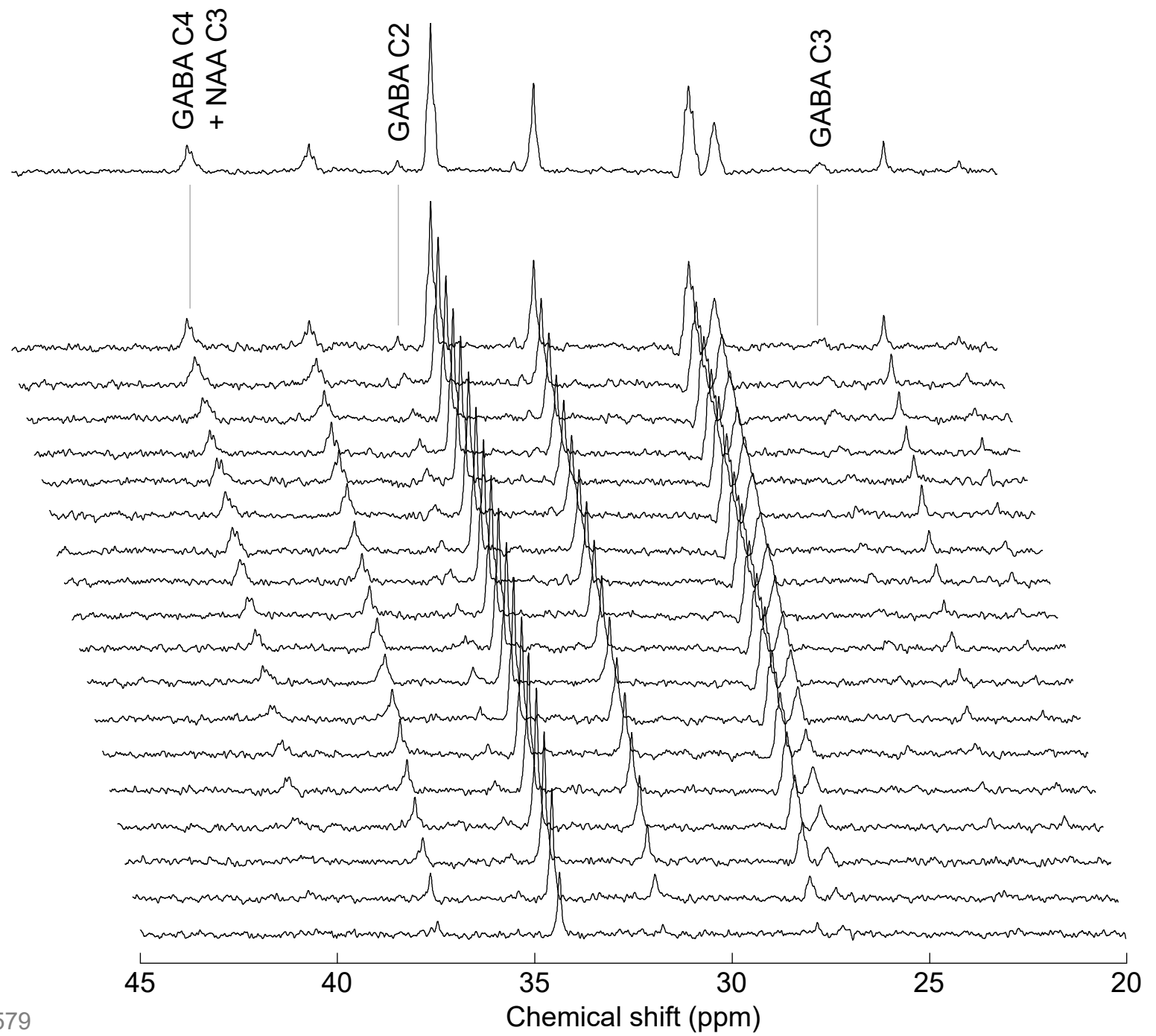
# Brain energy metabolism in diabetic rats



[1,6-<sup>13</sup>C]glucose administered to GK rats under α-chloralose (Girault *et al.* Neurotox Res 2019)

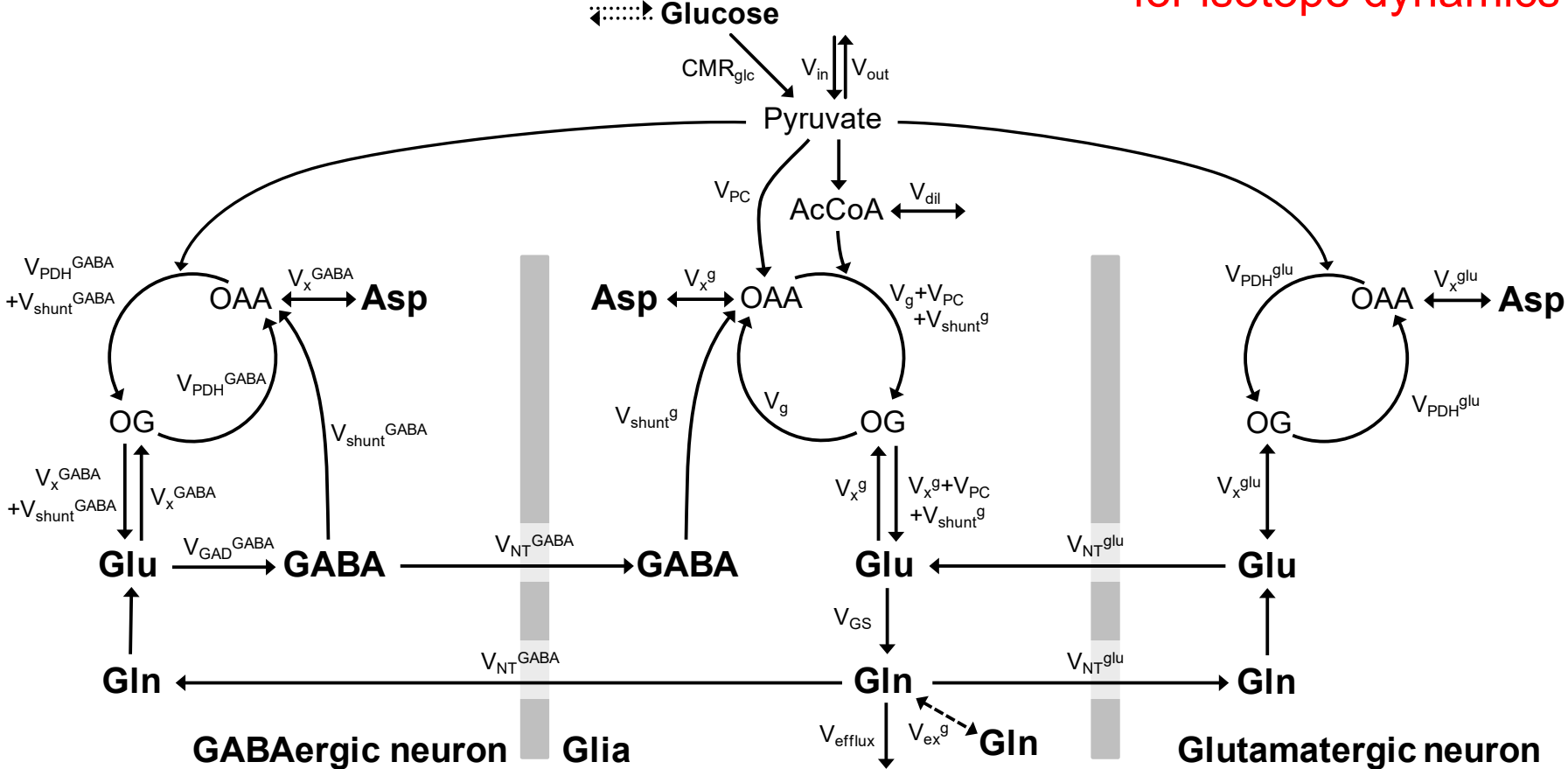
# Cortical energy metabolism upon somatosensory stimulation



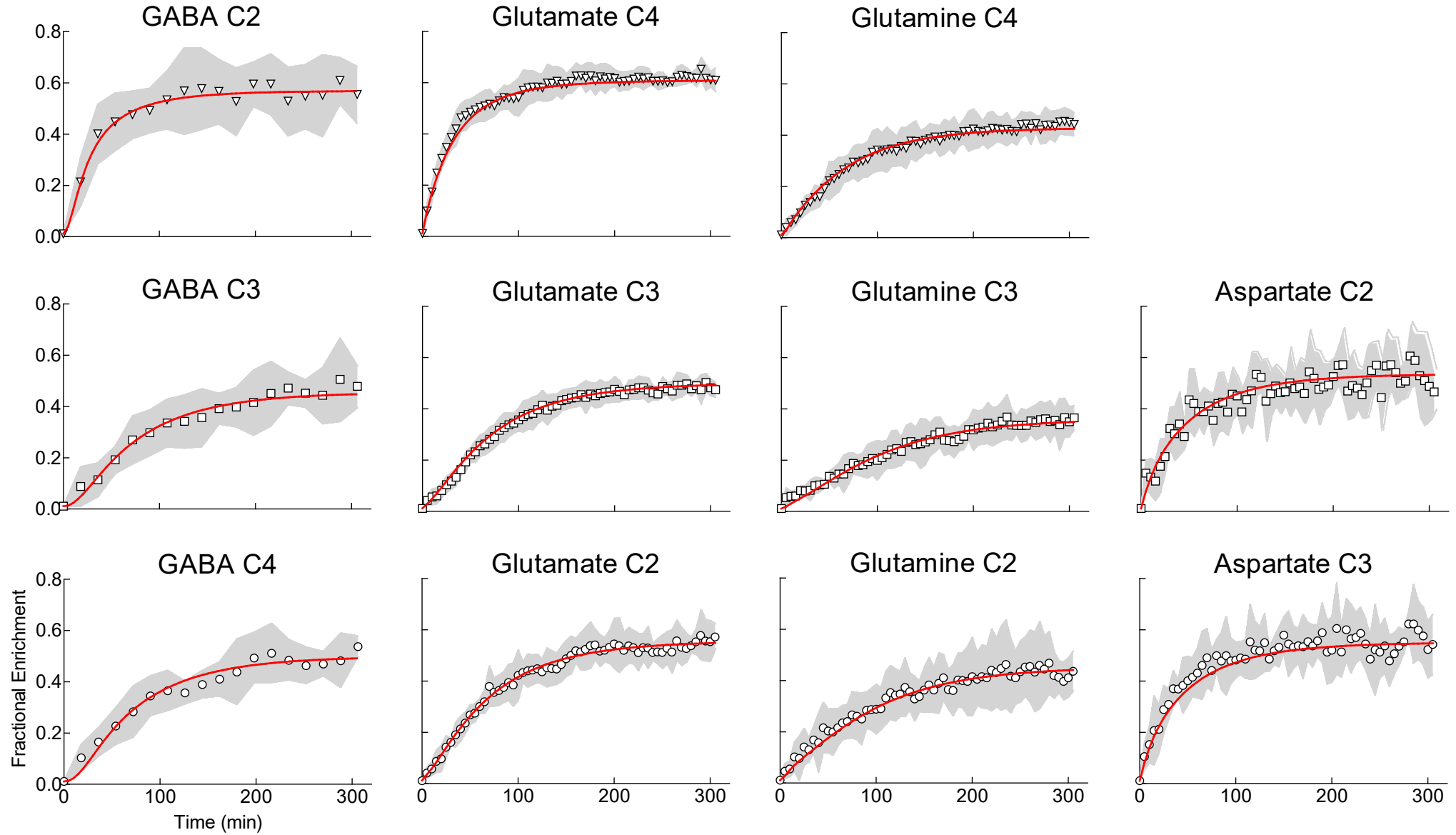


# 3-compartment model of brain glucose metabolism

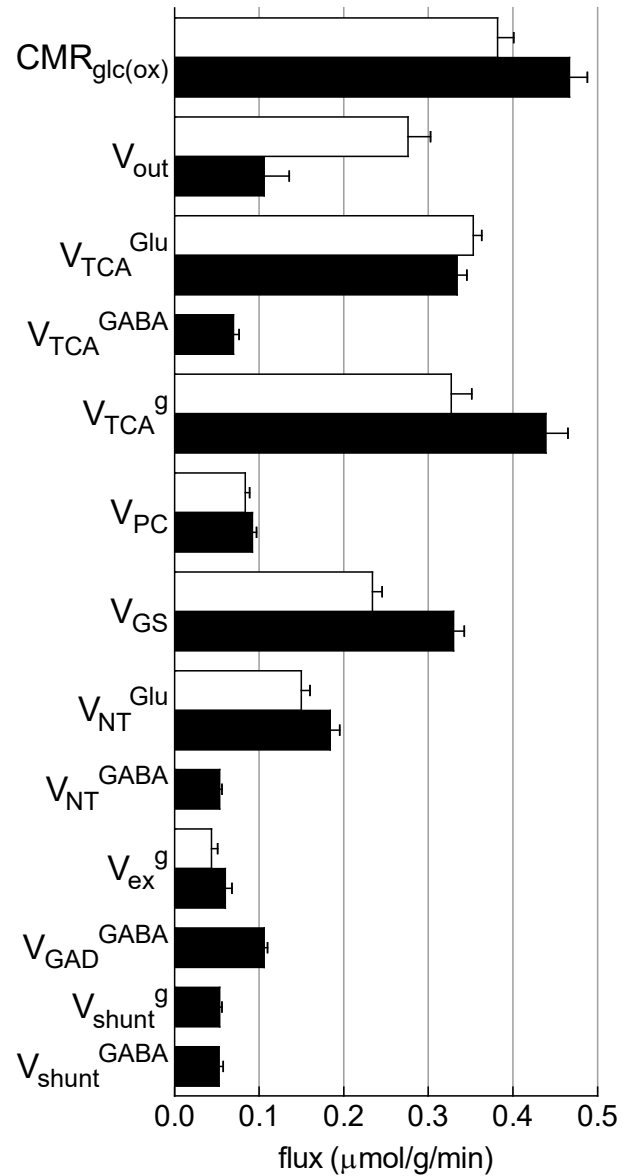
52 differential equations for isotope dynamics



# $^{13}\text{C}$ -enriched metabolites in the brain



# Metabolic fluxes from the 3 compartment model



Compartments	1	2	3
Experimental <sup>13</sup> C curves	3	8	11
Parameters to fit	4	8	12
Mean SD of fluxes	28% 7% V <sub>TCA</sub> 69% V <sub>X</sub>	10% 3% V <sub>TCA</sub> <sup>n</sup> 31% V <sub>X</sub> <sup>g</sup>	19% 3% V <sub>TCA</sub> <sup>n</sup> 103% V <sub>X</sub> <sup>g</sup>

# Isotopomer analysis

## Isotopologue

chemical specie that differ only in the isotopic composition

Hydrogen-related isotopologues of water:

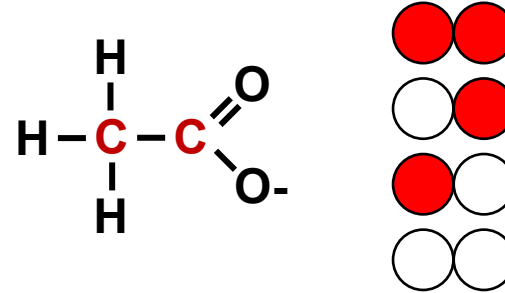
$^1\text{H}_2\text{O}$	light water
$^1\text{H}^2\text{HO}$	semi-heavy water
$^2\text{H}_2\text{O}$	heavy water
$^3\text{H}_2\text{O}$	tritiated water (super-heavy)

## Isotopomer

Malloy *et al.* (1988) *J Biol Chem* 263:6964

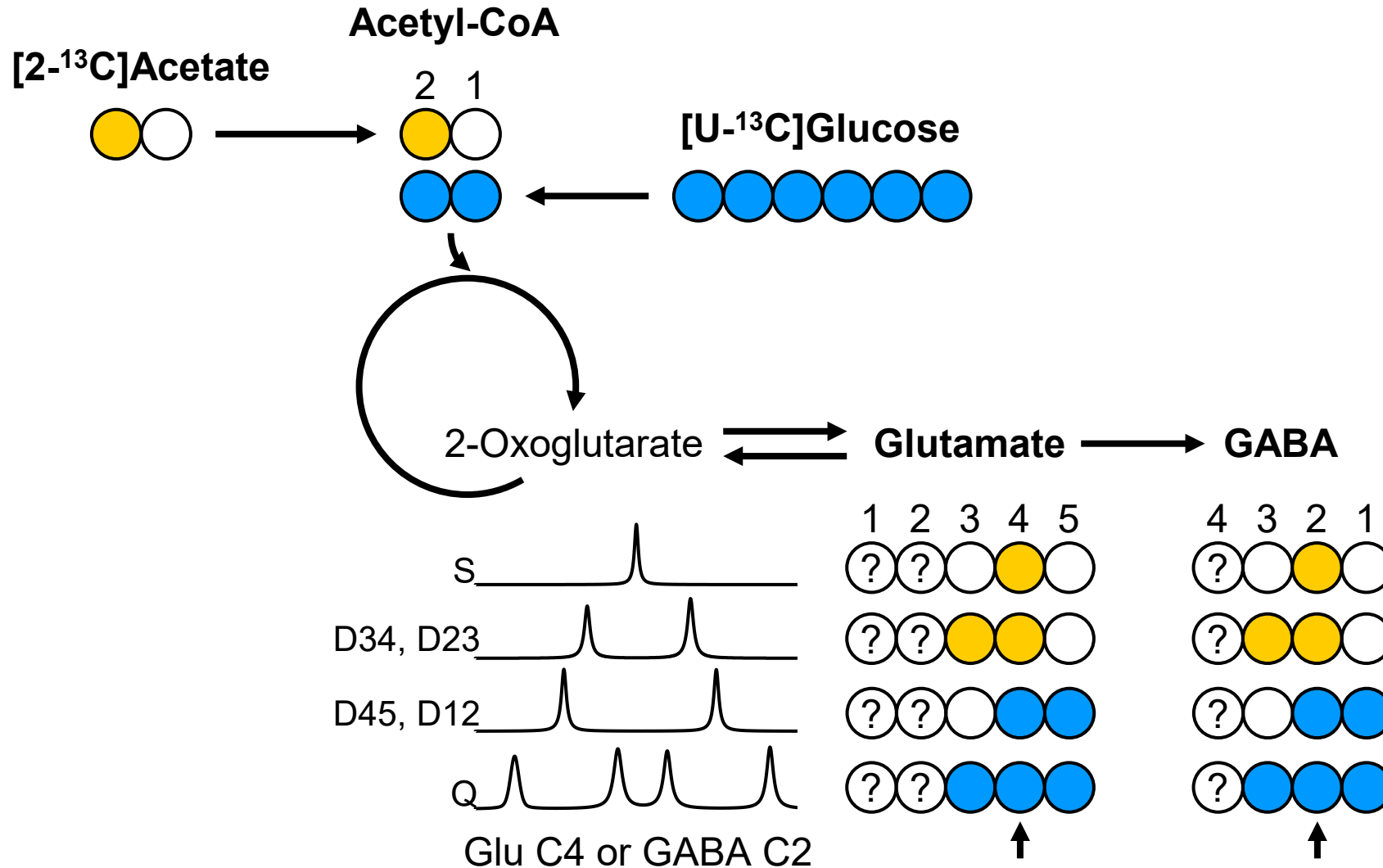
Isomers with same isotopic atom but different position

$^{13}\text{C}$  in acetate

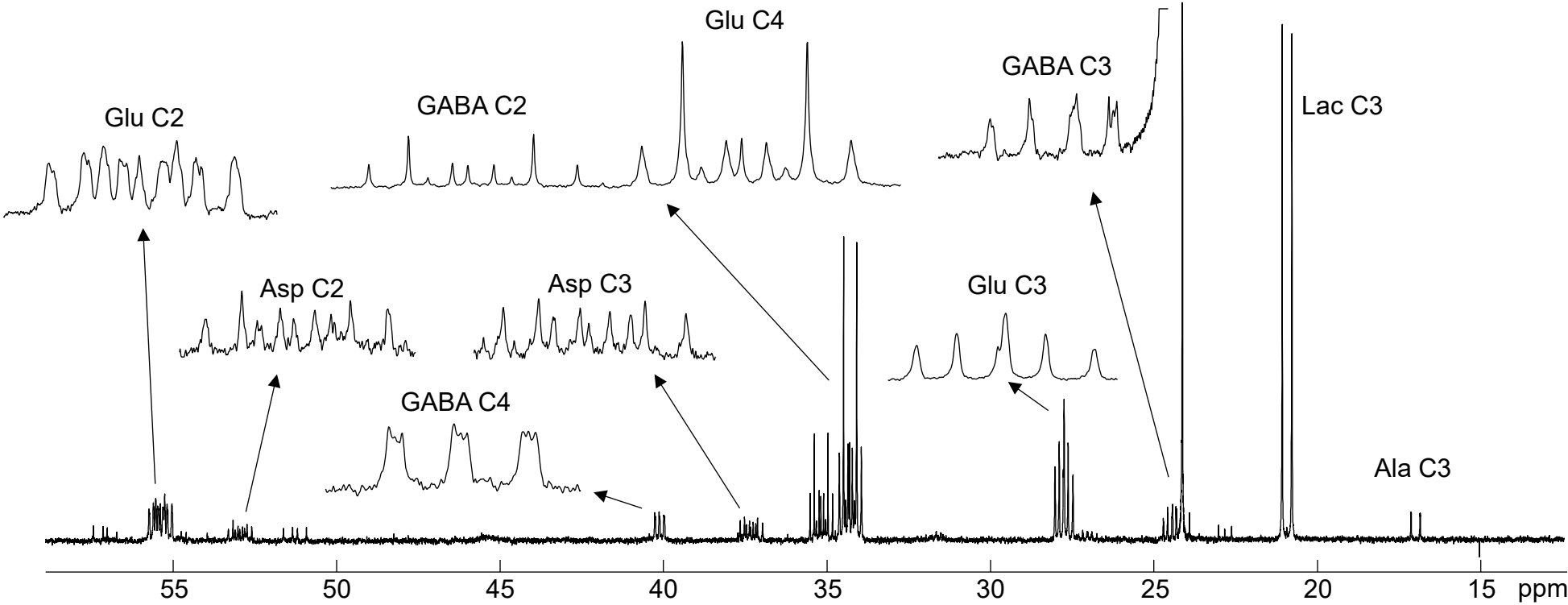


The positional enrichment of a carbon within a metabolite is the sum of all its isotopomer fractions where that carbon atom is labeled.

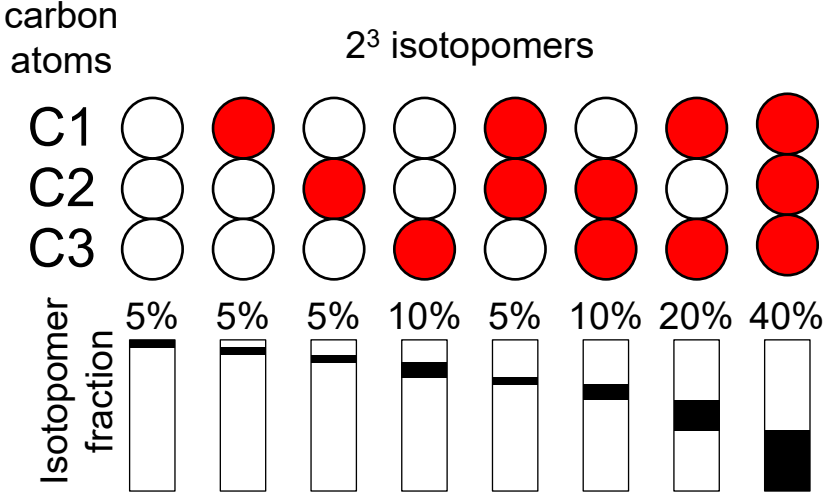
# TCA cycle: oxidation of [2-13C]acetate & [U-13C]glucose



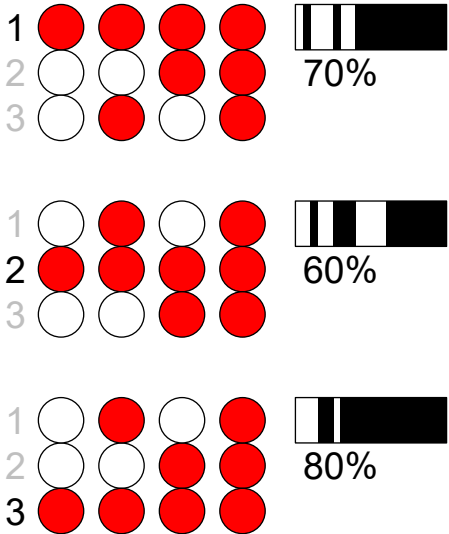
# Hippocampal slices with [2-<sup>13</sup>C]acetate & [U-<sup>13</sup>C]glucose



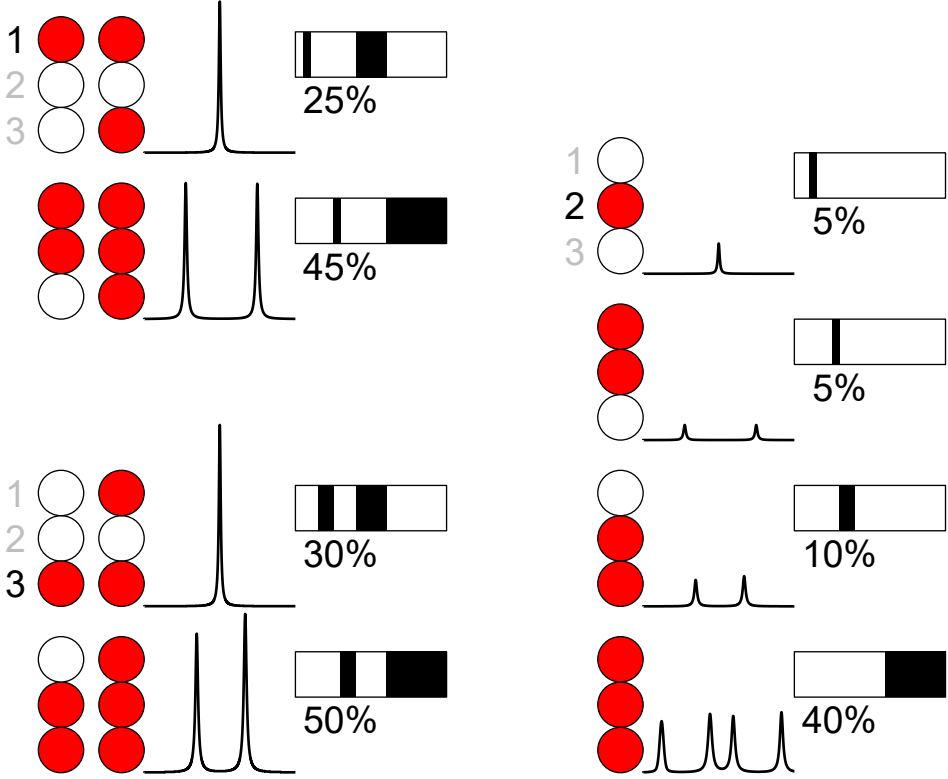
# Cumulated isotopomer: "cumomer"



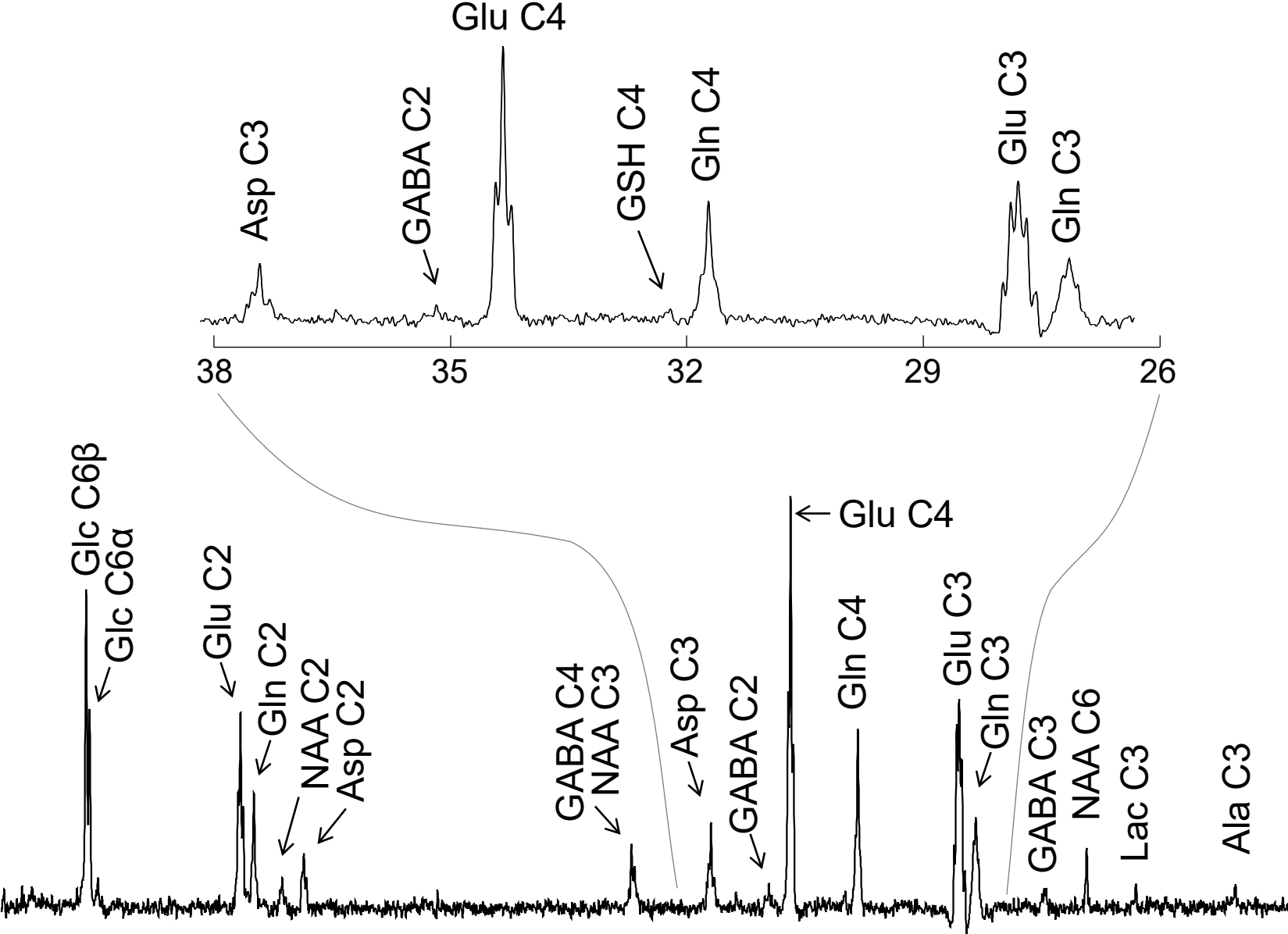
## Positional enrichments



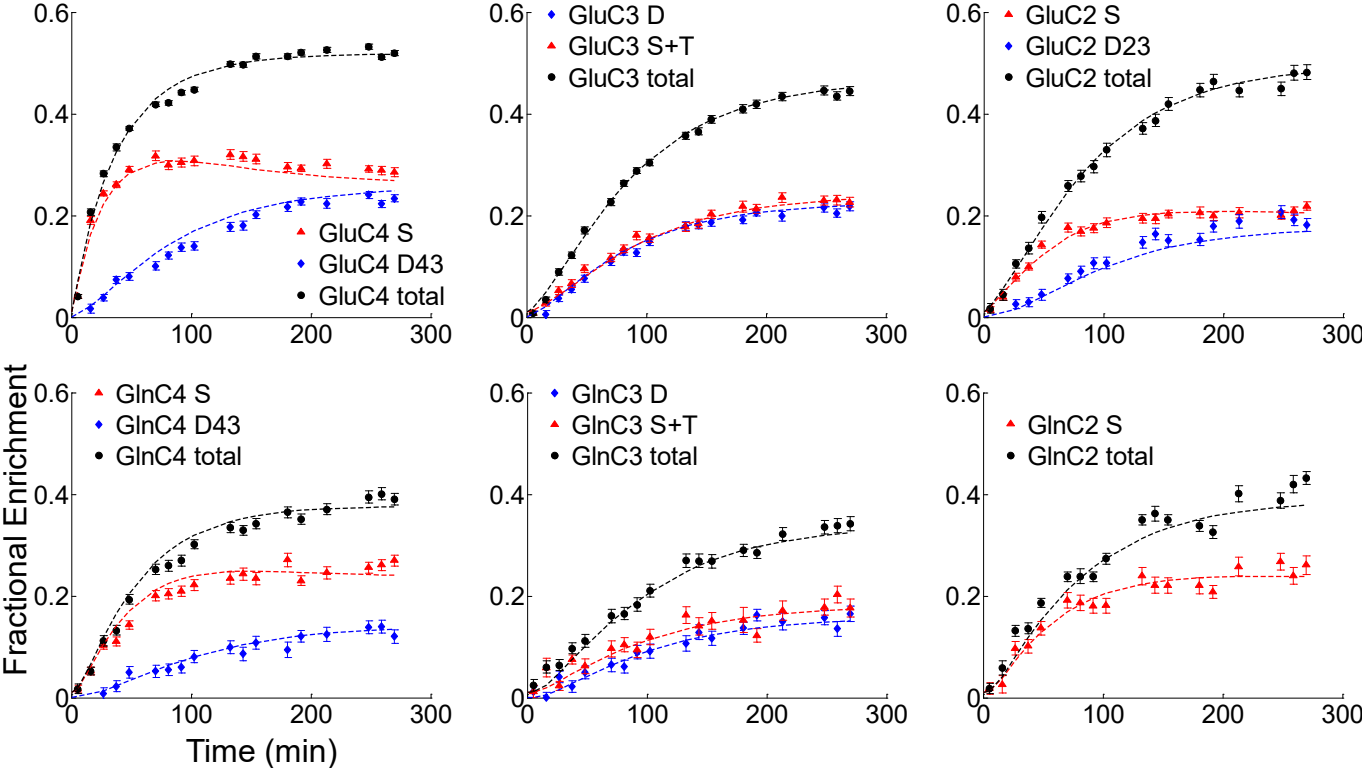
## Multiplets in $^{13}\text{C}$ MRS (cumomer fractions)



# Cumomers in addition to positional enrichments



# Cumomers in addition to positional enrichments

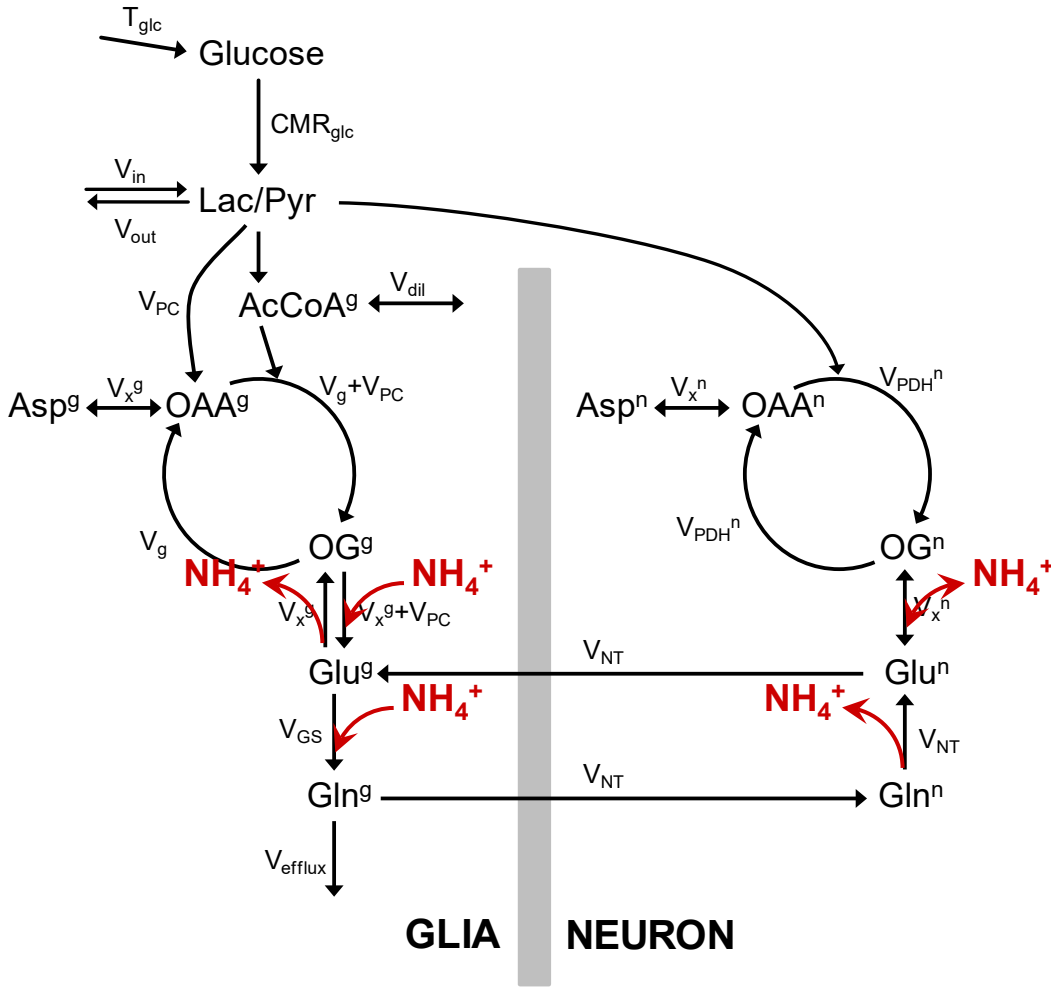
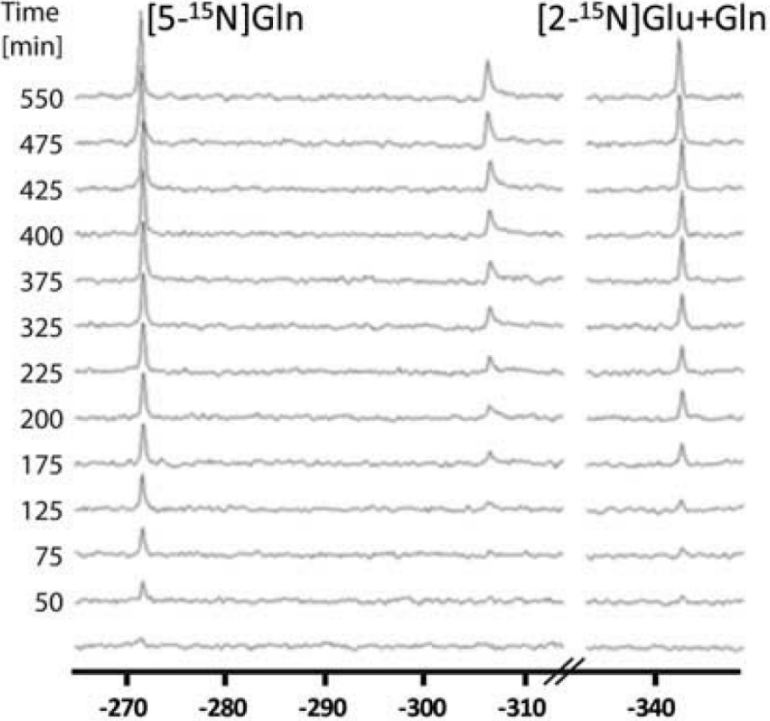


model	positional	positional + cumomer
Experimental <sup>13</sup> C curves	6	17
Parameters to fit	7	7
Mean SD of fluxes	9% 3% $V_{TCA}^n$ 18% $V_{NT}$	5% 3% $V_{TCA}^n$ 10% $V_{dil}$

# Non-stationary MFA: example with $^{15}\text{N}$ MRS

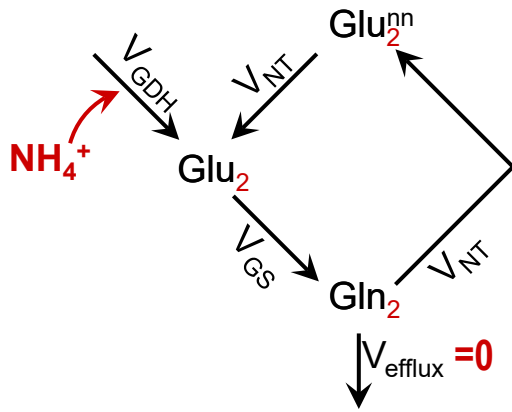
$^{15}\text{NH}_4\text{Cl}$  infusion

$^{15}\text{N}$  MRS in the rat brain

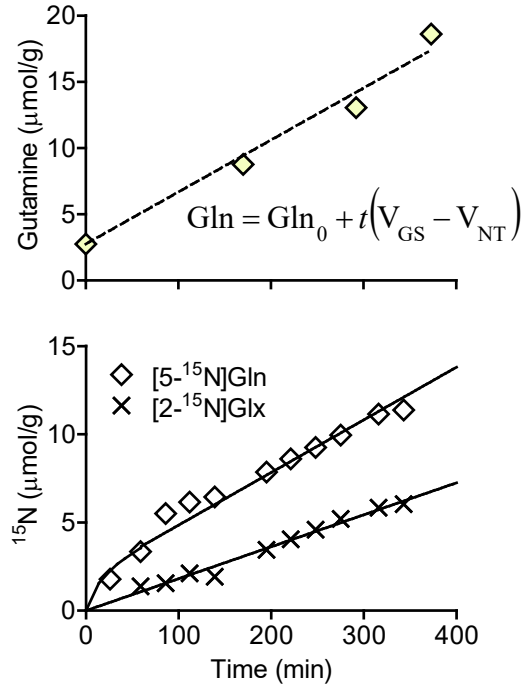
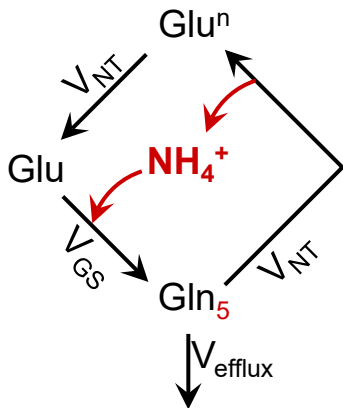


# Model for $^{15}\text{N}$ incorporation into glutamate & glutamine

## amination of C2



## amination of C5



## Mass balance equations

$$\frac{d \text{NH}_4^+}{dt} = V_{\text{in}}^{\text{net}} + V_{\text{NT}} - V_{\text{GS}} - V_{\text{GDH}} = 0$$

$$\frac{d \text{Glu}}{dt} = \frac{d \text{Glu}^{\text{n}} + d \text{Glu}^{\text{g}}}{dt} = V_{\text{GDH}} + V_{\text{NT}} - V_{\text{GS}} = 0$$

$$\frac{d \text{Gln}}{dt} = V_{\text{GS}} - V_{\text{NT}} - V_{\text{efflux}} \neq 0$$

## Isotope balance equations

$$\frac{d \text{ }^{15}\text{NH}_4^+}{dt} = V_{\text{in}}^{\text{net}} FE_{\text{pl}} + V_{\text{NT}} \frac{\text{Gln}_5}{\text{Gln}} - (V_{\text{GS}} + V_{\text{GDH}}) \frac{^{15}\text{NH}_4^+}{\text{NH}_4^+}$$

$$\frac{d \text{Gln}_2}{dt} = V_{\text{GS}} \frac{\text{Glu}_2^{\text{g}}}{\text{Glu}^{\text{g}}} - V_{\text{NT}} \frac{\text{Gln}_2}{\text{Gln}}$$

$$\frac{d \text{Glu}_2^{\text{n}}}{dt} = V_{\text{NT}} \left( \frac{\text{Gln}_2}{\text{Gln}} - \frac{\text{Glu}_2^{\text{n}}}{\text{Glu}^{\text{n}}} \right)$$

$$\frac{d \text{Glu}_2^{\text{g}}}{dt} = V_{\text{GDH}} \frac{^{15}\text{NH}_4^+}{\text{NH}_4^+} + V_{\text{NT}} \frac{\text{Glu}_2^{\text{n}}}{\text{Glu}^{\text{n}}} - V_{\text{GS}} \frac{\text{Glu}_2^{\text{g}}}{\text{Glu}^{\text{g}}}$$

$$\frac{d \text{Gln}_5}{dt} = V_{\text{GS}} \frac{^{15}\text{NH}_4^+}{\text{NH}_4^+} - (V_{\text{NT}} + V_{\text{efflux}}) \frac{\text{Gln}_5}{\text{Gln}}$$

# $^{13}\text{C}$ MFA *in vivo*: take home message(s)

! Models as simple  
• as possible.

! Collect as much data as possible  
• from the system to be modelled.

! Document model  
• assumptions.

! Model comparison and  
• reliability/validity testing.

! Matching carbons between each  
• molecule and its precursor.

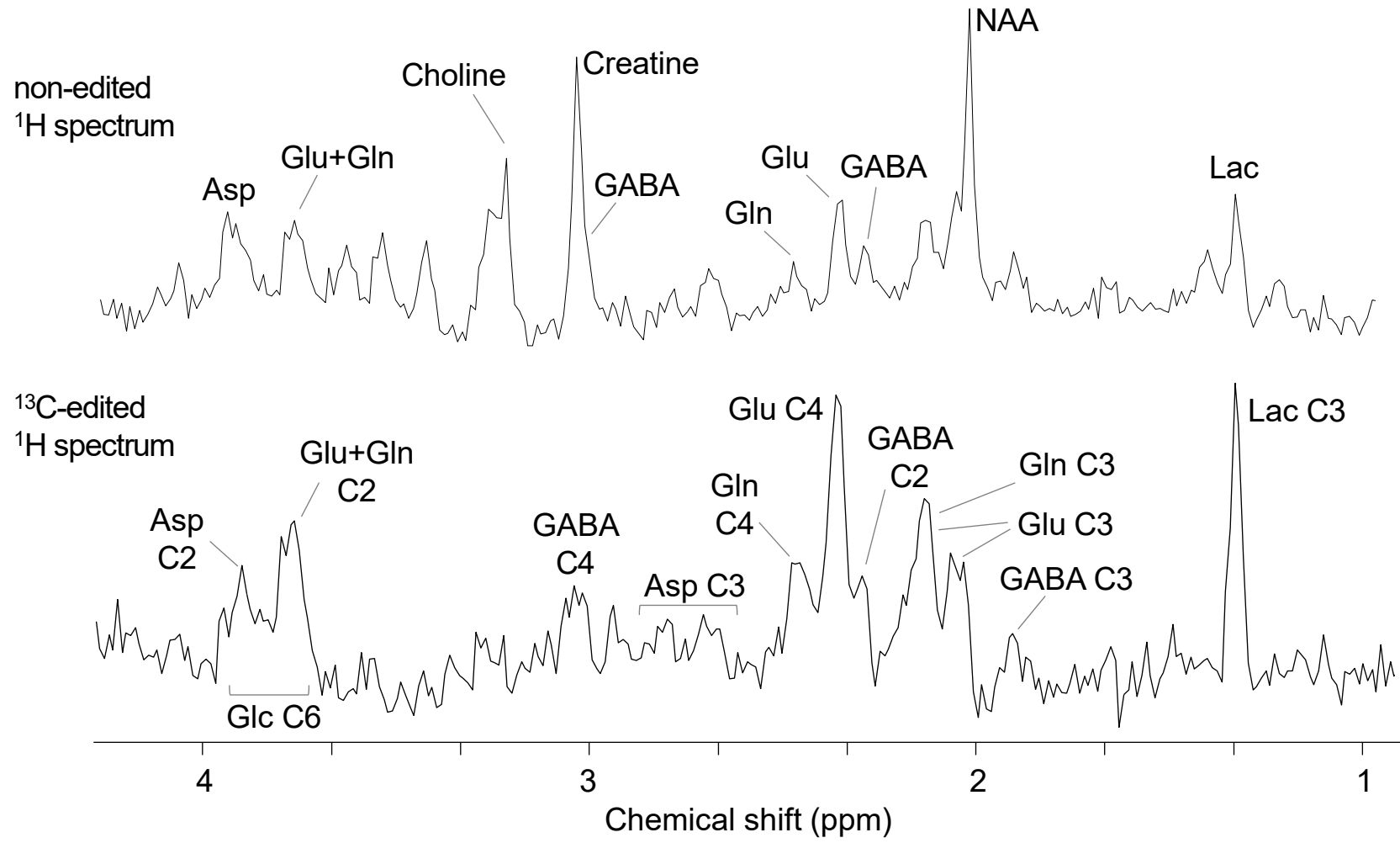
! MFA models contain  
• no information about  
regulatory mechanisms

Little  
predictive  
power!

**Metabolic  
Control  
Analysis (MCA)** ✓

# SNR is an important limitation

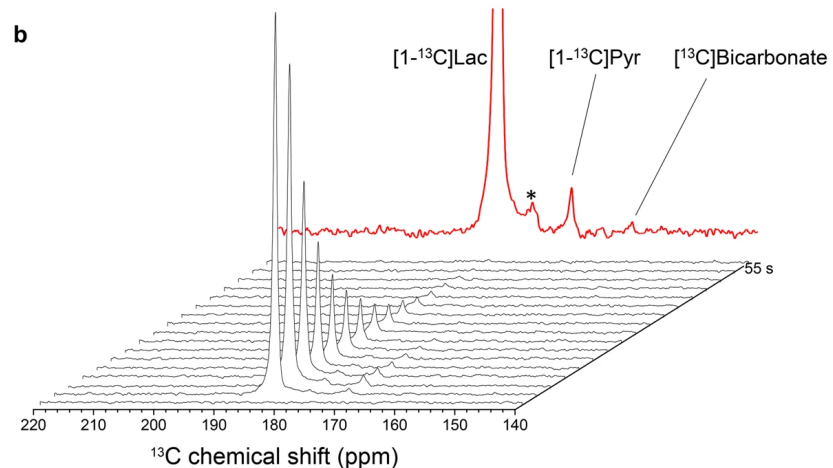
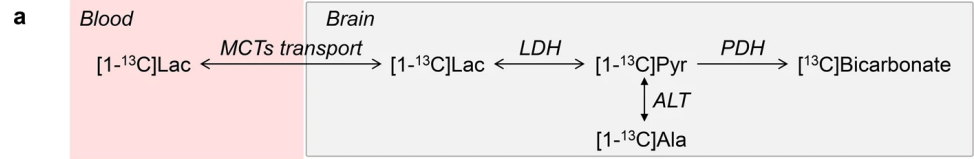
## Indirect detection of $^{13}\text{C}$



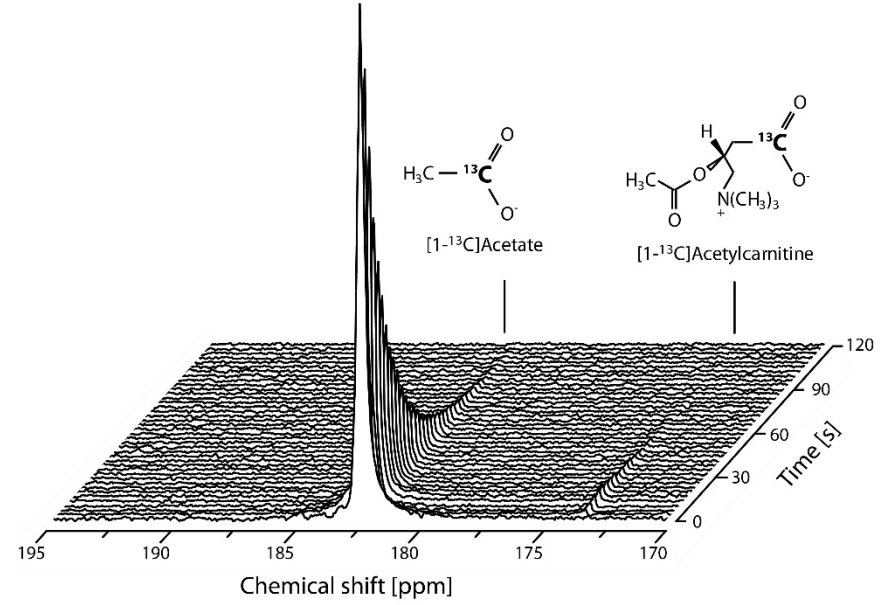
Lizarbe *et al.*, Int J Obes 2019 (see discussion in Duarte, Neurochem Res 2025)

# SNR is an important limitation

## Hyperpolarized MRS



Hyacinthe *et al.* Sci Rep 2020

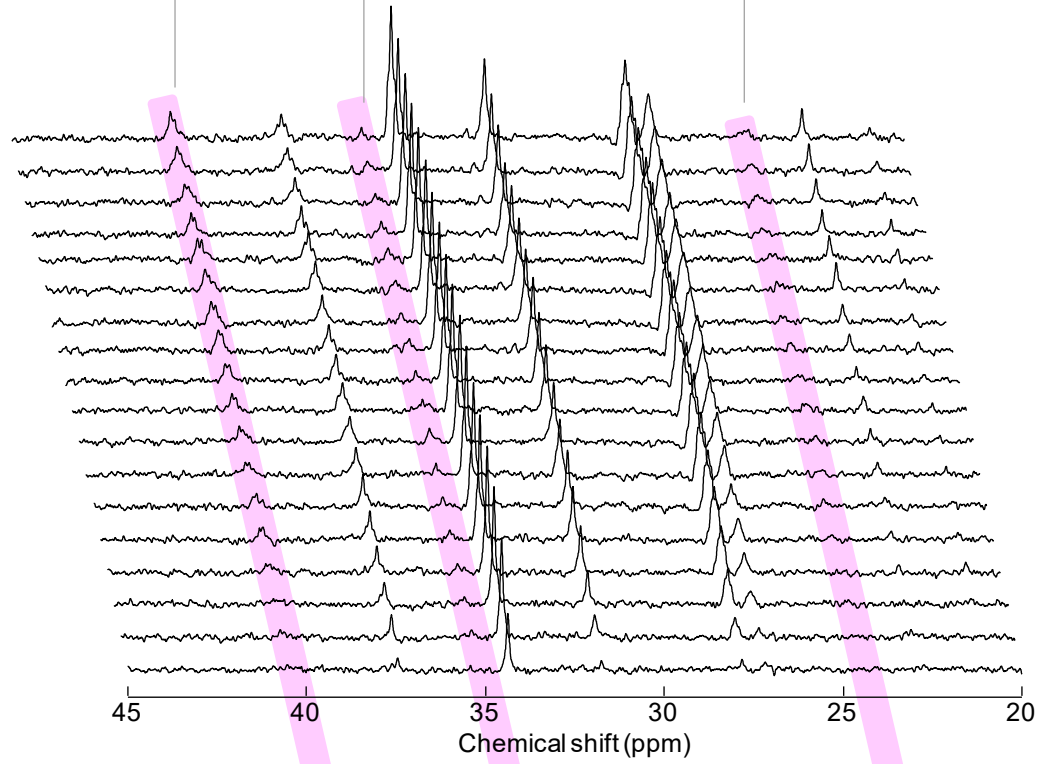
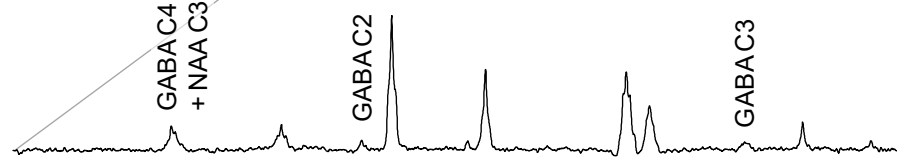
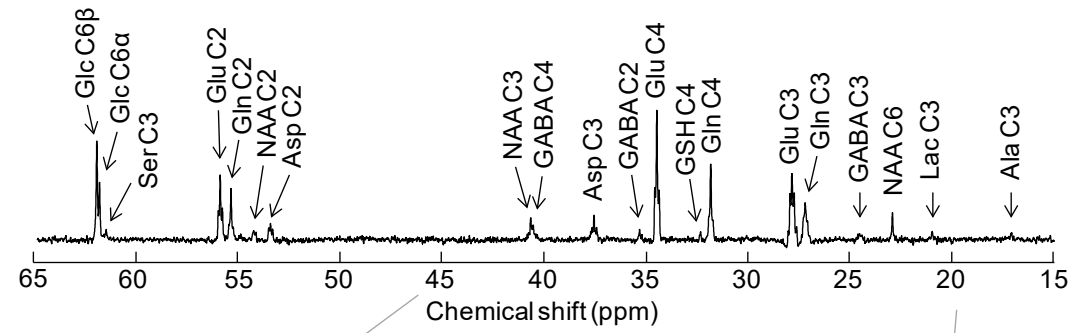


Bastiaansen *et al.* BBA 2013

# $^{13}\text{C}$ NMR spectra from the rat brain *in vivo*



**GABA is 4-fold smaller than glutamate**



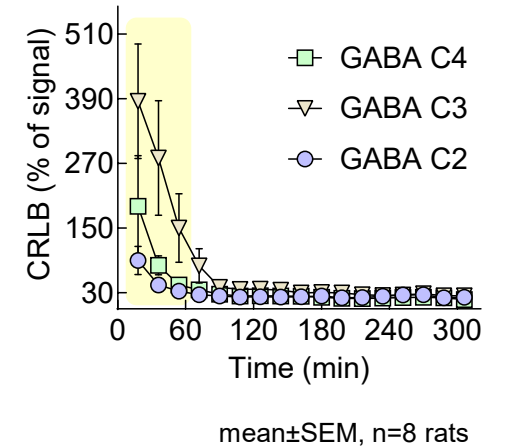
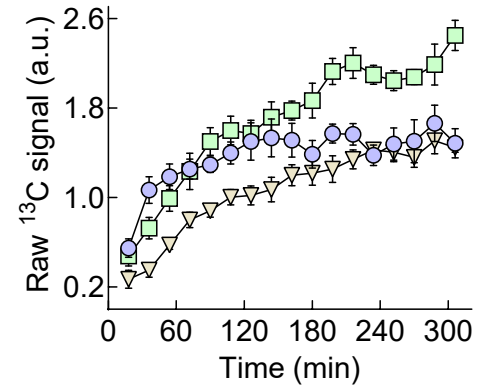
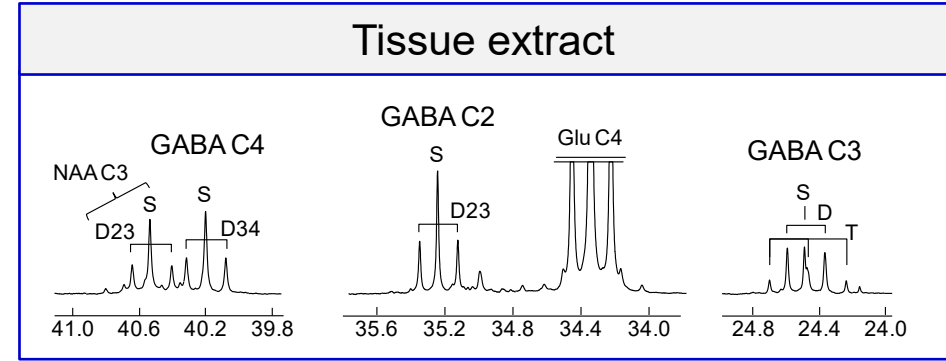
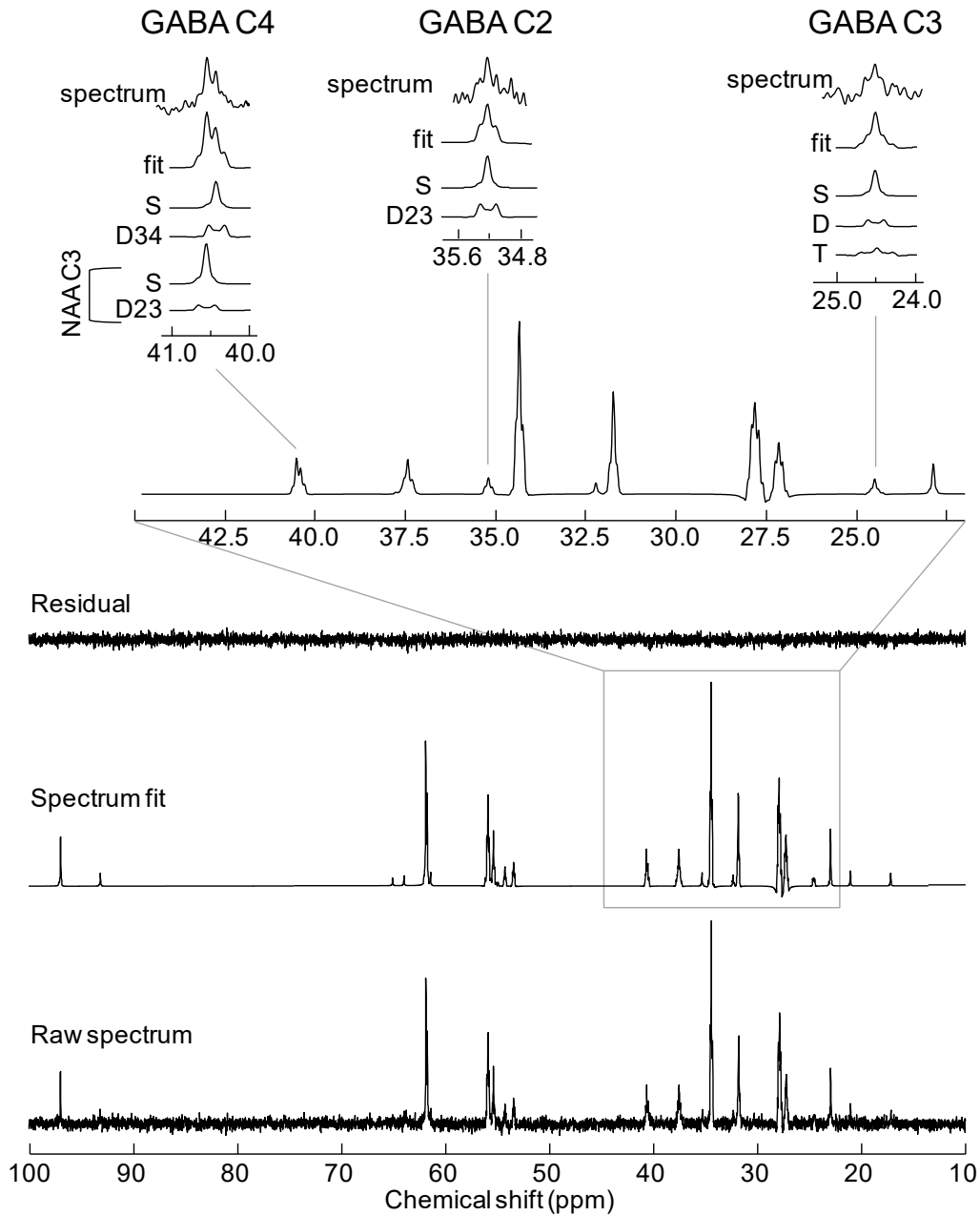
## Bottom:

4x128 scans with TR of 2.5 s (21.3 min)  
Lorentzian–Gaussian apodization (lb=5, gf=0.08, sbs=0.02)

## Top:

Sum of 3 last spectra (~1-h)  
Shifted Gaussian (gf=0.08, gfs=0.02)

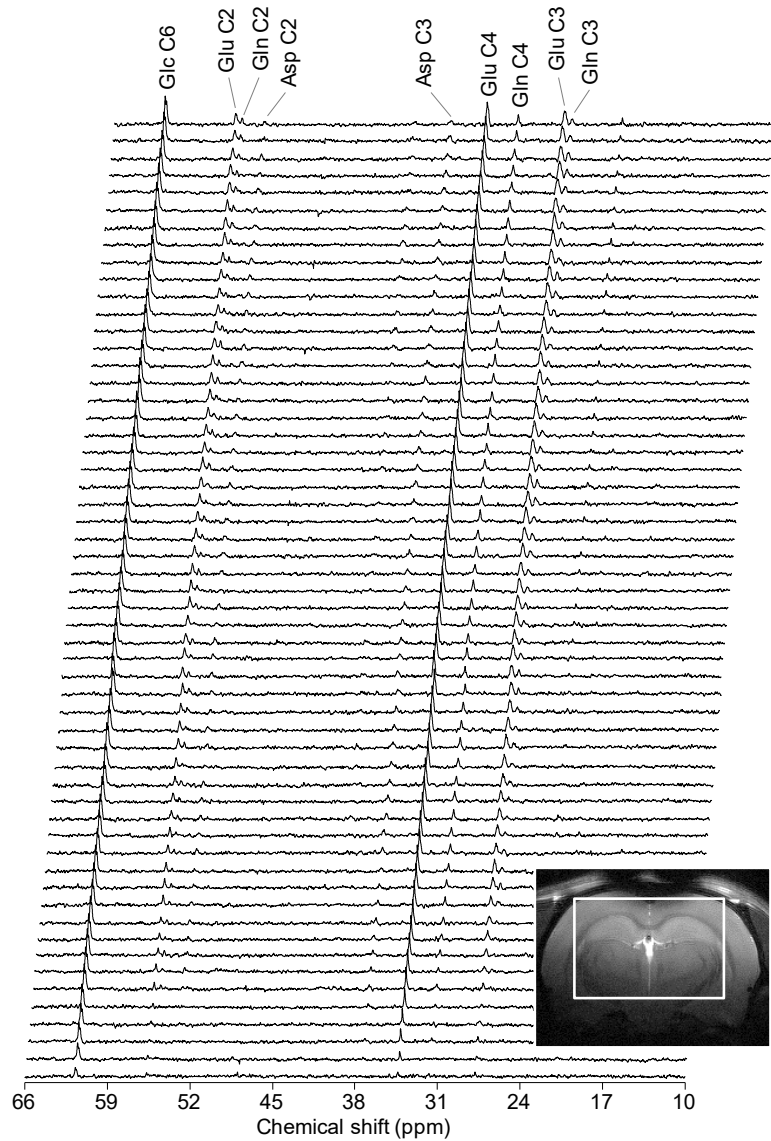
# Quantification of $^{13}\text{C}$ NMR spectra using LCModel



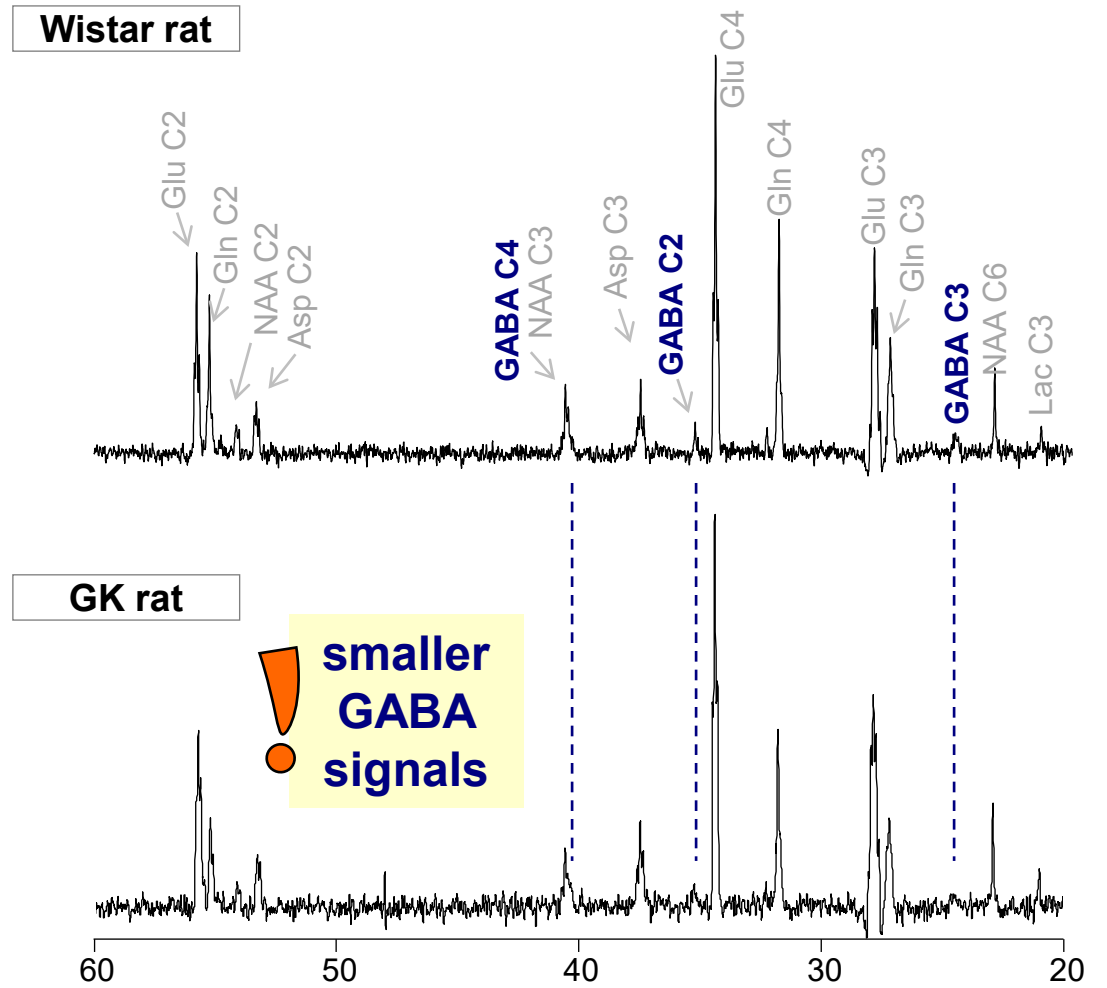
**Inaccurate estimation of GABA during the 1<sup>st</sup> hour !**



# Brain metabolism in diabetic rats

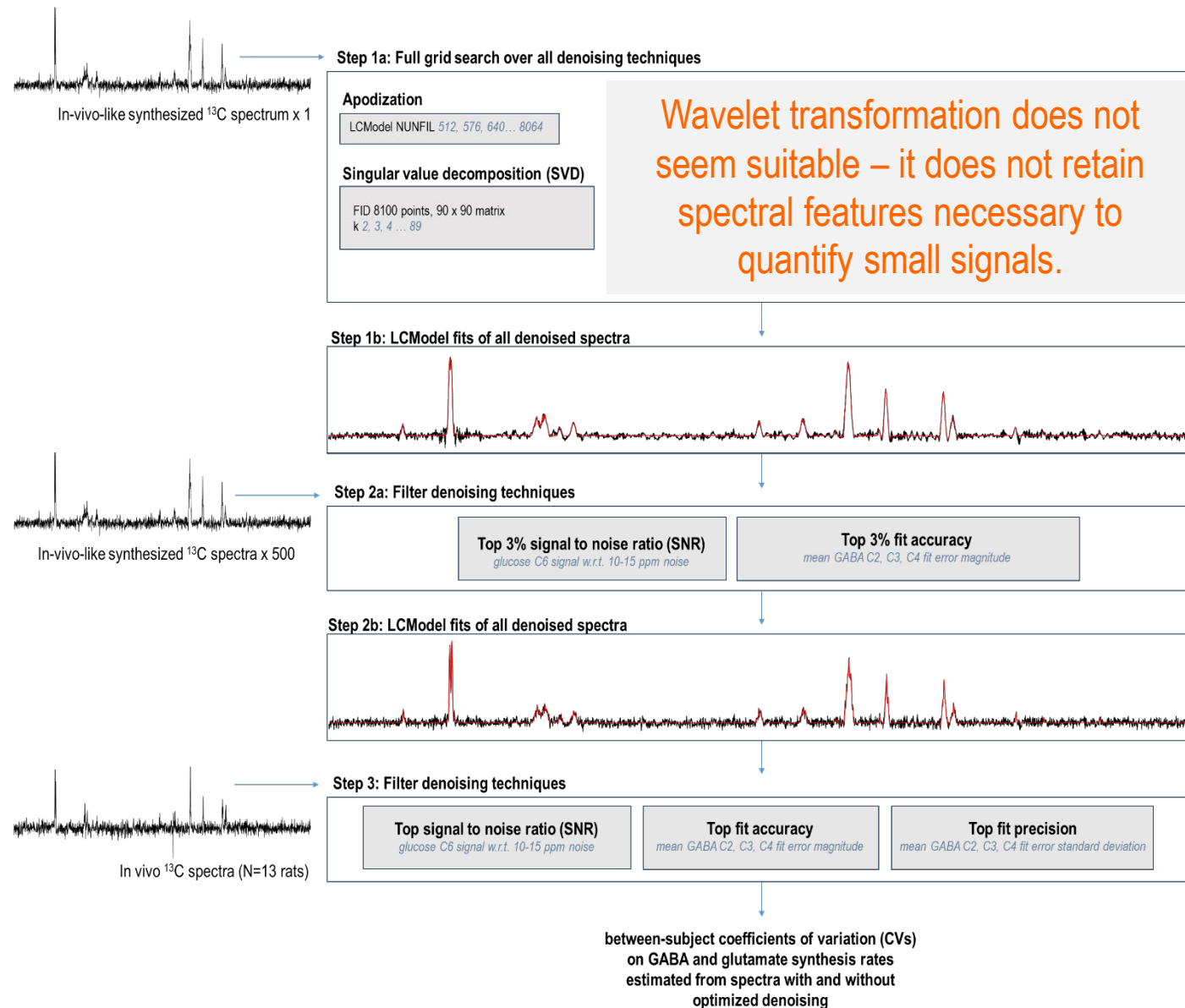


**Left:**  
 128 scans with TR of 2.5 s (5.3 min) for about 5 h  
 Lorentzian–Gaussian apodization (lb=7, gf=0.08, sbs=0.02)



**Right:**  
 Sum of 3 last spectra (~1-h)  
 Shifted Gaussian (gf=0.08, gfs=0.02)

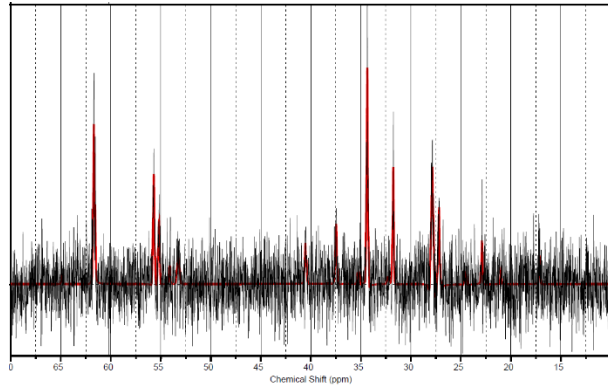
# Can spectral denoising techniques improve the reliability of GABA estimates by $^{13}\text{C}$ NMR spectroscopy?



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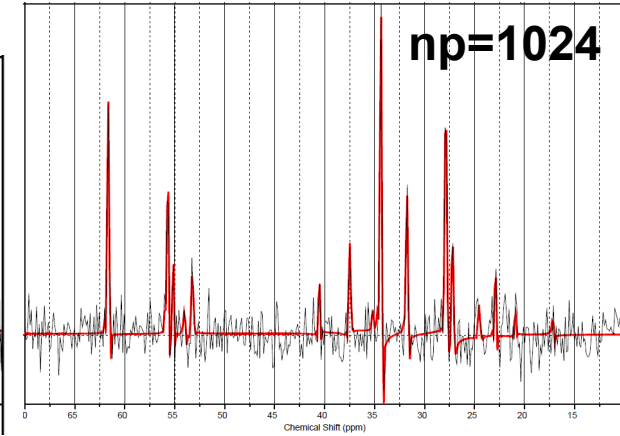
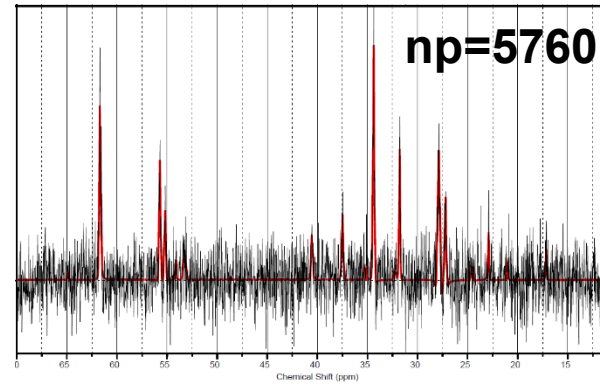
- Apodization/FID truncation impacts analysis of multiplets.
- SVD seems promising for increasing precision but needs to be further tested *in vivo*.

# Results Analysis 3: *in vivo* application – fit of 128 scans

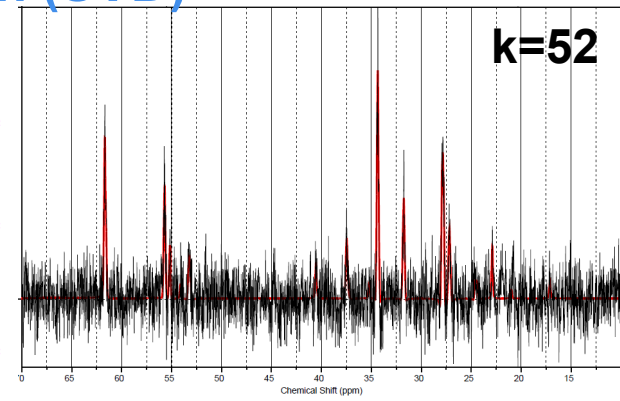
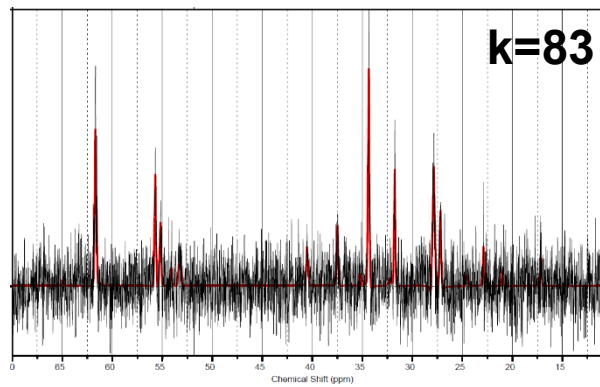


Original spectrum  
(np=8065)

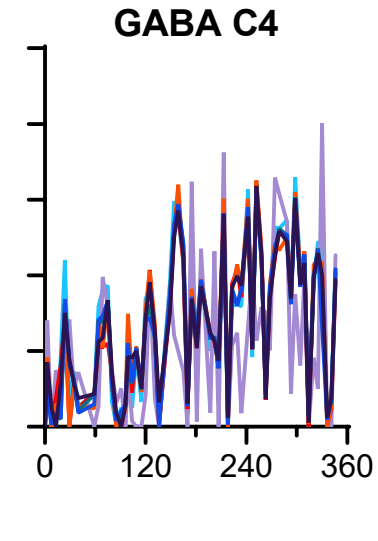
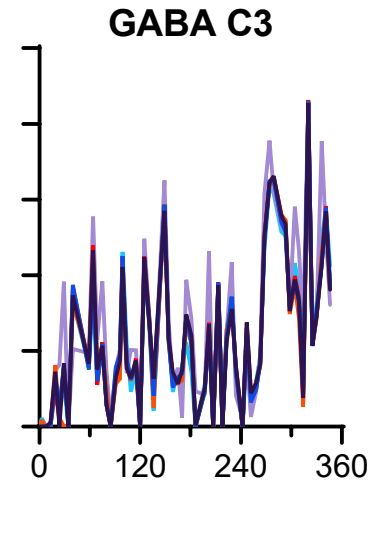
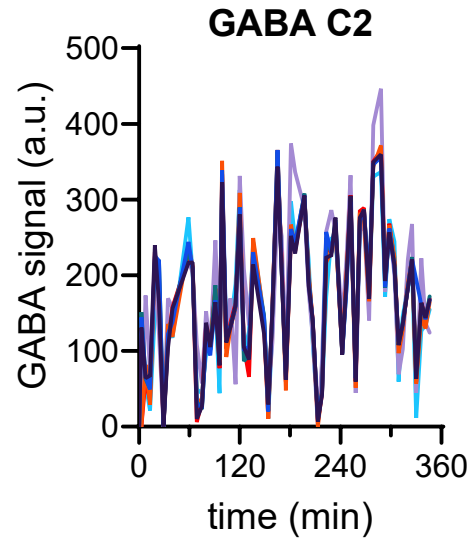
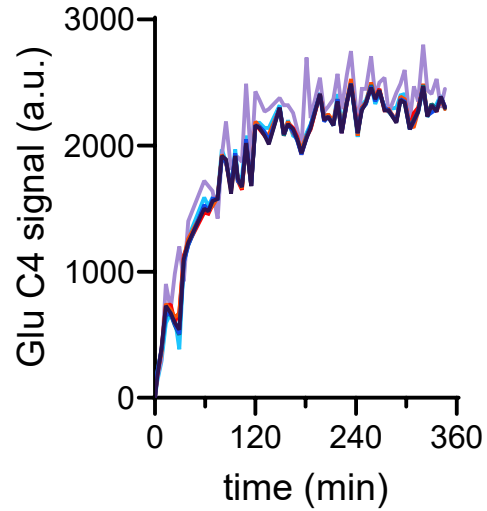
## FID truncation



## Singular value decomposition (SVD)



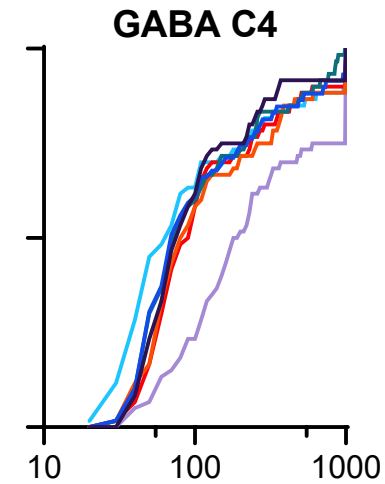
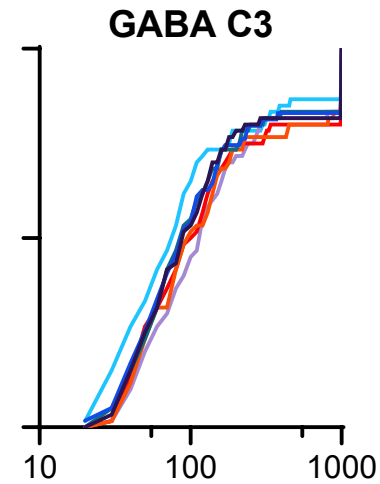
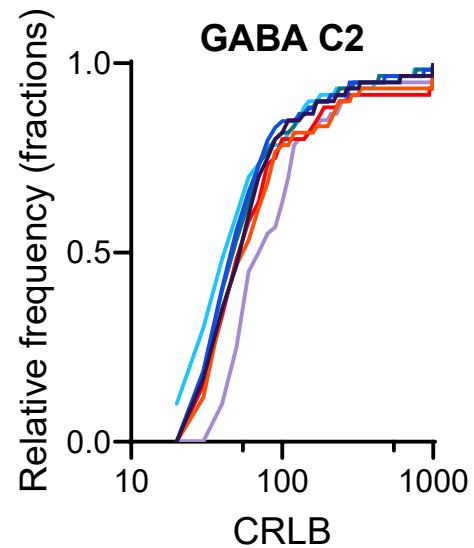
# Results Analysis 3: *in vivo* application – fit of 128 scans



- None
- SVD\_90\_80
- SVD\_90\_83
- AD\_4352
- AD\_5760
- AD\_1024
- SVD\_52

There's no noticeable improvement in signal fit result

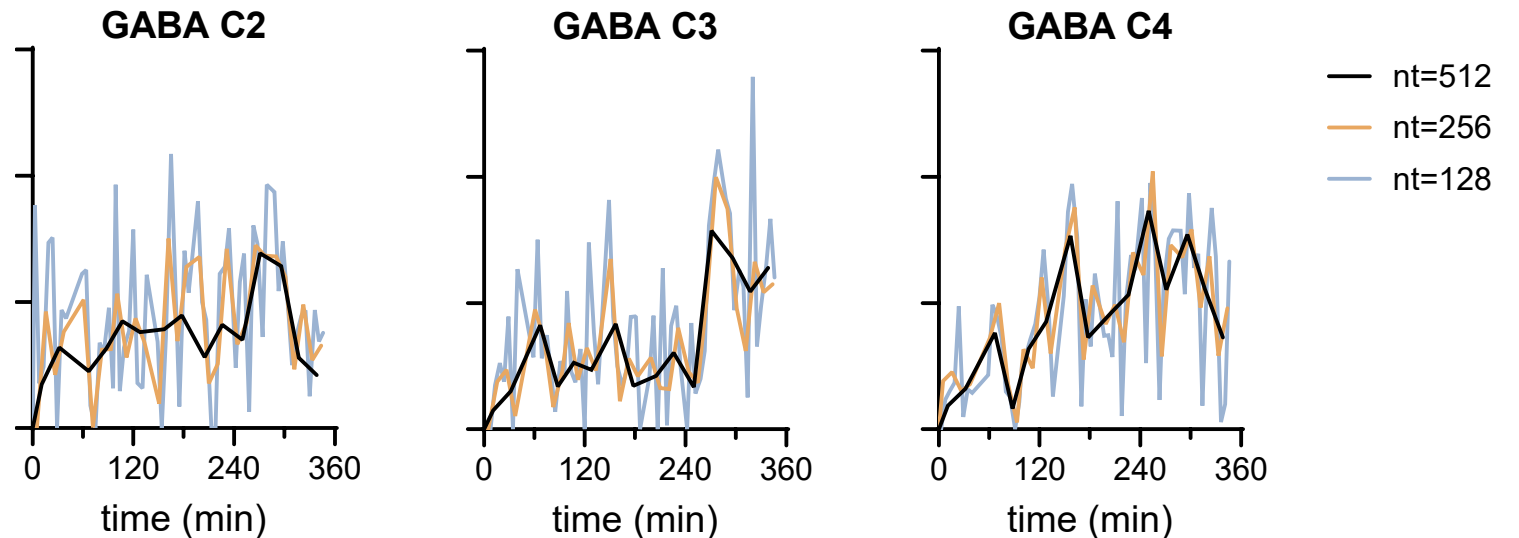
General decrease of CRLBs with SVD\_52



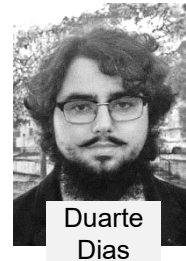
# Conclusion

- Noise reduction is essential, but not straightforward
- Testing SNR enhancing methods that take in account that *spectral features are consistent over time* might be interesting.
  - (1) PCA of redundant data (Veraart et al. NeuroImage 2016)
  - (2) Stationary Wavelet Denoising for Multiple Similar scans (Song et al., JMR 2024)
- Denoising using neuronal networks

The best approach remains  
the sum of more spectra  
= improve SNR at acquisition



## Diabetes & Brain Function unit



Postdoc on  
metabolism  
NMR+MS  
coming soon



@DiaBrainFun